HERBERT WESTREN TURNBULL

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Herbert Westren Turnbull was born in Wolverhampton on 31 August, 1885. He was educated at Sheffield Grammar School and from there went to Trinity College, Cambridge, as a scholar. After being placed Second Wrangler in 1907 he obtained a First in Part II of the Mathematical Tripos, and in 1909 he won the Smith Prize. A conventional academic career, which began with appointments at Cambridge and Liverpool, was interrupted by his move to Hong Kong, where from 1913 to 1915 he was acting Warden of the Church Mission Society hostel of Hong Kong University whilst at the same time holding the post of a lecturer in Mathematics. On his return to England he taught at Repton for three years and subsequently served as an H.M. Inspector of Schools, which had been his father's profession. He resumed academic work on being appointed Regius Professor of Mathematics at St. Andrews University in 1921. In 1950 he retired from this chair with the title of Professor Emeritus.

Turnbull was a mathematician of the classical school. He made substantial contributions to the theory of invariants. During his student years at Cambridge he was introduced to this fascinating branch of algebra when the memories of the great British and Continental masters of the subject were still fresh. His enthusiasm for invariants, and in particular for the "symbolical calculus" of Clebsch and Aronhold, remained with him throughout his life. It was unfortunate for him that already in the 1920's the fashion in algebraical research had drastically changed, and his original work on invariants did not receive the recognition which it would have found two decades earlier. However, his *Theory of Determinants, Matrices and Invariants*, first published in 1928, is still one of the most readable text-books on these topics. The theory of matrices also engaged his interest over a long period, and he introduced it into the undergraduate syllabus at a time when the subject was not widely taught at British universities. His *Introduction to the Theory of Canonical Matrices* (with A. C. Aitken) (1932) gives a very useful account of matrix algebra and some of its applications.

Turnbull was a prolific author. His style was lucid, and in his own writings and those of his pupils he insisted that no mathematical jargon was permitted to mar the purity of English prose. His approach to mathematics was concrete and formal in the sense that he sought to solve problems by an effective formalism rather than by a conceptual analysis of the underlying mathematical structures. His topics were algebraical, but he was fond of presenting them against a geometrical background. His

memoir, "The Geometry of Matrices" [*Phil. Trans. Roy. Soc.* (A), 239 (1942)] is a typical example of this point of view. Altogether his mind was more intuitive than deductive, and he never became reconciled to the modern trend towards abstraction.

Apart from his contributions to classical algebra Turnbull will be remembered as an authority on the history of mathematics. He always included a few lectures (and one examination question) in his First Year Course at St. Andrews, and his charming little book *The Great Mathematicians* (1929) provided an excellent text-book for his students. He turned to more substantial historical researches with a study of James Gregory. His findings were published in the *James Gregory Tercentenary Volume* (1938). After his retirement from the chair at St. Andrews he devoted himself exclusively to a major work on 17th century mathematics, which had been requested by the Royal Society, under the title *The Correspondence of Isaac Newton*. Turnbull lived to see the publication of the first two volumes of this monumental and beautifully produced work.

Among the distinctions he received were the Fereday Fellowship at St. John's College, Oxford, for the period 1919–26 and the election to the Royal Society in 1932. His great merits for mathematics in Scotland were acknowledged by the awards of the Keith Medal and the Gunning Victoria Jubilee Prize of the Royal Society of Edinburgh and by the LL.D. degree of St. Andrews University.

In his lectures Turnbull favoured an informal style, sometimes relying on improvisation, which was frequently prompted by a research problem that was occupying him at that moment.

He was very sensitive to the formal beauty in mathematics, and this perhaps provides the link with his love for music. His talents as a pianist far exceeded the attainments of the average amateur. It was a great pleasure to play chamber music with him, where his experience and genuine musicianship showed themselves to full advantage.

His other pastime was mountaineering. He was a member of the Alpine Club and an intrepid climber, often without a guide. In the Scottish Highlands he pioneered several difficult ascents. He was a member of the Scottish Mountaineering Club and, at one time, its president. Nearer his home opportunities for practice were provided on the cliffs of St. Andrews Bay. He discovered fourteen ways up "The Maiden Rock". The mastery of the "Rock and Spindle" was not exactly part of the mathematical syllabus, but many a student experienced on this striking formation his first thrill of rock climbing under the guidance of his professor of mathematics.

In his relations with students, colleagues and friends and indeed with everybody who came into contact with him, Herbert Turnbull showed inexhaustible kindness and patience. It is inconceivable that a harsh word

should have fallen from his lips or that he should have harboured hostile feelings towards any human being. He was deeply religious in the widest sense of the word. His faith permeated his every action and undoubtedly was the reason for his serenity, whether it was in the face of physical danger on a mountain or on a more trivial level when he had lost the thread during a lecture, or when at an amateur performance of chamber music, in which he was taking part, one of the players (usually not himself) had blundered and the ensemble became unsteady.

In 1911 he married Ella Drummond Williamson, daughter of Canon H. D. Williamson. There was one son of the marriage. In their home at St. Andrews they extended hospitality to countless students and friends. At these gatherings Mrs. Turnbull was a gracious and lively hostess. The inevitable shyness of the younger guests was overcome by drawing room games, but the highlight of the evening, for those who could appreciate it, was the performance on two pianos by Professor and Mrs. Turnbull. Their playing, highly musical and exquisitely blended, was a beautiful expression of a harmonious partnership.

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