OSWALD VEBLEN

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Oswald Veblen, whose sudden death at his holiday home in Maine occurred in August 1960, was not only a very important figure in the mathematical world at large, but he was a most influential member of this Society. A member of the Society for many years, he served on the Council in the year 1928-29, when his counsels were of the utmost value to the Society at a critical time in its history, and later, in 1939, he became an honorary member. An enthusiastic Anglophile, his interest in the Society was only equalled by his interest in British mathematicians, and a list of those who are in his debt for encouragement and advice in their younger days would include the names of nearly every senior mathematician in the country.

It is not possible to do justice to Veblen's services to mathematics in a notice of this kind, since not only must the writer deal with his published contributions to mathematics, but he must try to give proper recognition to the work which he did behind the scenes to organise the science of mathematics in the United States, and much of the vital information on this is only to be found in the records of various institutions which are not accessible to British mathematicians. Veblen's life, indeed, has coincided with the growth of American mathematics from its humble origins to its present imposing stature which has hardly a rival in the world today, and in this development no one has played a more decisive part than Veblen. He was eight years old when the American Mathematical Society was founded, with 16 founder members, and very little influence indeed; today it has some 7,500 members, and enjoys a world-wide reputation, and in this growth Veblen played one of the leading parts.

Veblen was born in Iowa in 1880. His paternal grandparents had emigrated in 1847 from Norway, and after a period in Wisconsin had settled on a prairie farm in Minnesota, where they brought up twelve children in the hard conditions of life in the North West at that period in American history. One of these children was T. B. Veblen, who became a famous economist and social theorist; another was Andrew Anderson Veblen, who became Professor of Physics at the University of Iowa, and was the father of Oswald. After attending various public schools in Iowa City, Veblen studied at the University of Iowa, where he took his A.B. degree in 1898, after which he spent a year as an assistant in his father's laboratory. This was followed by a year at Harvard, where he took his A.B. degree in 1900, whence he proceeded to the University of Chicago to work for his Ph.D., where he came under the influence of Bolza, Maschke and E. H. Moore. He obtained his doctorate in 1903 for a dissertation on a system of axioms of Euclidean geometry, in which he followed the ideas of Pasch

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and Peano rather than those of Hilbert and Pieri. After taking his doctorate, he remained for two years in Chicago as Associate, and then moved, in 1905, to Princeton, which was to be his home for the rest of his life. For the first five years at Princeton he was Preceptor; from 1910 to 1926 he was an ordinary professor, and then for another six years he was H. B. Fine Research Professor. In 1932 he became a Professor at the Institute for Advanced Studies, and he was largely responsible for building up the splendid school of mathematics there.

In 1908 he married Elizabeth Richardson, the sister of the late Sir O. W. Richardson, who was at that time Professor of Physics at Princeton. She proved a magnificent helpmate to him, and many will recall with pleasure the hospitality they received from the Veblens, first in Battle Road, and later on their estate outside Princeton; and even more will remember the enchanting visits that the Veblens paid to their friends in their own homes on their frequent visits to Europe.

Veblen's thesis gave a clear indication of where his interest in mathematics lay, namely, in the foundations of the subject. The work he did in many papers on geometry, but above all in his famous treatise on Projective Geometry written in collaboration with J. W. Young, did more than anything else to get rid of the fuzziness which used to surround most of geometry, and turn the subject into an exact science. It is true, of course, that others have also done this, but sometimes their work has seemed, though logically satisfactory, to be rather detached from the main body of the subject. This was certainly not true of Veblen, whose account of the foundations is quite straightforward and leads on naturally to the development of the subject. The same might be said of Veblen's work on Differential Geometry and on Spinors. For Veblen had the gift of grasping a simple scheme on which the whole of projective or differential geometry could be based, so that the subject was given a sound foundation without any excursions into quasi-philosophy. The same clear insight was revealed in his Colloquium Lectures on Analysis Situs, which in a sense could be said to have launched modern combinational topology on its triumphant course. It is difficult after so many years, during which so much has happened in the mathematical world, to realise how important Veblen's work was at the time. It has all taken its place in the history of mathematics, and one tends to take it for granted. But it certainly did a very great deal for mathematics, especially American mathematics, and his services to the subject are probably of more fundamental value than the discovery of a number of splendid theorems which would bear his name.

In fact, service to mathematics was the keynote of Veblen's life. A very great deal of his energies went into "back room" work, in which he expended himself in building up American Mathematics into its present position of prestige. As a young man in Princeton he worked hand in hand with H. B. Fine, magnificently supported by President Woodrow Wilson,

to build up the great school at Princeton. The American Mathematical Society, too, is everlastingly in his debt. The period of his Presidency of the Society in 1923-24 was a time of crisis in the Society. The Society's position, as a private organisation with no official standing, left it in a very insecure position, and, with the increasing output of research papers which it was called on to publish, its financial position was unsound. A certificate of Incorporation under the code of the District of Columbia was obtained during Veblen's Presidency. This put the Society in a stronger position to help itself, but it did not, by itself, solve the many practical problems which faced the Society. Veblen set about dealing with these problems with vigour and efficiency. He analysed the problems and set up committees to consider the various issues, such as membership, finance, and so on. When certain legal difficulties arose in collecting endowments, Veblen threw himself into the problem and devised ingenious ways to enable corporations to contribute. Altogether Veblen and Coolidge succeeded so well that by 1930 the Society had raised its endowments by $70,000 and they both, deservedly, received the special thanks of the Society. To Veblen it used the words: "Your interest in every phase of the work, your willingness to aid at any personal sacrifice, your enthusiasm, imagination and pertinacity were ever at work for the cause." But, it might have been added, Veblen's concern was not just to make the American Mathematical Society great for its own sake, but because he felt that a strong society was necessary for the well-being of American mathematics.

Veblen was always primarily concerned with the ultimate good of mathematics, and this concern was soon to result in another and greater step forward. In 1932, shortly after the Institute for Advanced Studies had been founded, Dr. Flexnor, the Director, met Veblen in the street one day, and asked him how he thought he should go about building up a school of mathematics. Veblen expounded in his usual lucid way the ideas that had been forming in his mind, and did it so effectively that the next morning he received an invitation to become the first mathematical professor in the Institute. He accepted at once, and threw himself with enthusiasm into the task of putting his ideas into practice, with results that the whole world knows. It was not part of Veblen's plan that the professors at the Institute should necessarily themselves produce great streams of papers, and Veblen himself did not write much after his appointment. But by his availability to all who wanted to consult him, by his encouragement of the younger generation, and by his flair for detecting outstanding talent, and even genius, to say nothing of his ability to keep in touch with recent developments even when old age came upon him, his contribution to the greatness of the school can be measured. He remained a faithful member of the Institute even after his retirement, until eventually increasing blindness became too much of a handicap.

One other thing must be said. When a scientist has great organising ability and decides to put it at the service of his subject, he is apt to make enemies, who misunderstand his motives and accuse him of seeking position and power. Veblen, in spite of his authority in American Mathematics, made few enemies and all came to recognise what he had done for mathematics. His modesty, and his great charm, endeared him to all his colleagues, and few mathematicians can have been better loved than he. His election as President of the International Congress of Mathematicians held at Harvard in 1950 was as much an indication of the affection of his colleagues as of their regard for him as a mathematician. It was characteristic of him that he began his Presidential Address on that occasion with the words: "In taking the chair today I feel that I am just acting as deputy for my friend, George Birkhoff, whose untimely death has kept him from performing this duty. It was he who could have best welcomed the mathematicians of the world both on behalf of his University and on behalf of the American Mathematical Society." One can only say that had Professor Birkhoff lived until 1950, our American colleagues would have had a hard task in deciding between them.

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