GEORGE NEVILLE WATSON

R. A. RANKIN

George Neville Watson was born on 31 January, 1886, at Westward Ho! in Devon, and died in Leamington Spa on 2 February, 1965. His father, George Wentworth Watson, was master at the United Services College, Westward Ho!, but moved to London before Rudyard Kipling went there as a boy, and so is not mentioned in *Stalky and Co.* The son was educated at St. Paul's School, London, which he entered as a Foundation Scholar in 1898. In October 1904 he went up to Trinity College, Cambridge, with a Major Scholarship. He was a pupil of R. A. Herman of Trinity College and of A. Berry of King's College. In 1907 he was classed as Senior Wrangler in Part I of the Mathematical Tripos; in the same class list appear the names of H. W. Turnbull, A. V. Hill and J. R. Wilton in the second, third and fifth places. At that time the First Class in Part II of the Tripos was still divided, and he was placed in the Second Division of that class in 1908 (Turnbull was placed in the First Division). The following year he obtained a Smith's Prize and in 1910 he became a Fellow of Trinity College. He held his Fellowship until 1916, but in 1914 he went to University College, London, as an Assistant Lecturer and was promoted to an Assistant Professorship the following year.

Professor R. S. Heath, who was the first Professor of Mathematics in Birmingham University, retired in 1918 and Watson was asked to succeed him. He remained in Birmingham from 1918 until his retirement at the age of 65 in 1951. When he came the Mathematics Department was still housed in the original Mason's College building in Edmund Street (only recently abandoned), although the new buildings had already been set up in Edgbaston over two miles to the south. In due course the department moved in its entirety to Edgbaston, although there was an awkward intermediate stage when lectures were given in both places. Watson set up an Honours School and widened the scope of mathematical studies; at first the department comprised both pure and applied mathematics, but in 1937 the department divided and Watson became Professor of Pure Mathematics.

In 1925 he married Elfrida (Freda) Gwenfil Lane, the daughter of the late Thomas Wright Lane. I am indebted to Mrs. Watson and her son George for help in compiling this notice.

Watson became a member of this Society in 1907 and gave it devoted service in the years that followed. He served on the Council from 1918 to 1936 and from 1937 to 1946, being Honorary Secretary from 1919 to 1933.

[JOURNAL LONDON MATH. SOC., 41 (1966), 551–565]

He was Honorary Editor from 1937 to 1946, President from 1933 to 1935, and Vice-President from 1935 to 1936. He was awarded the De Morgan Medal in 1947. For his outstanding contributions to mathematics he received awards from many other societies and institutions. In 1912 the Royal Danish Academy presented him with their Gold Medal for the work contained in his paper [**17**] and in 1946 he received the Sylvester Medal of the Royal Society, of which he was elected a Fellow in 1919. He received honorary doctorates from Dublin and Edinburgh and was an Honorary Fellow of the Royal Society of Edinburgh.

Until his last few years in Birmingham he took a very active part in university affairs, and his advice was sought on both academic and administrative matters. He was instrumental in improving superannuation conditions for university staff and, being an expert on railway time-tables, he was given the task of making the lecture time-table for the whole university. He never had a secretary, nor did his department have any annual grant, but, in return for his services as a compiler of a workable lecture programme, the Registrar's office used to duplicate any material needed for circulation to students.

For many years Watson was actively involved in the work of the Northern Universities Joint Matriculation Board and the Oxford and Cambridge Board, both as an Examiner and Moderator. His devotion to this Society has been mentioned, but he also took a more than usual interest in other societies of which he was a member and he was deeply concerned for their welfare. The Mathematical Association was one of these, and he was its President in 1932-33; his first paper [**1**] was published in the *Mathematical Gazette* and he was a regular contributor throughout his life. Another was the Edinburgh Mathematical Society, although it is doubtful whether he ever attended a meeting of that body.

He had a great admiration for his friend and co-author, the late Sir Edmund Whittaker, but, although he doubtless received several invitations, he only visited Scotland twice, once in June 1939 to receive his Honorary LL.D. from Edinburgh University, and in July 1914 to attend the Napier Tercentenary Congress. He used to say that he feared to make a third visit (and this no doubt explains why he was not admitted in person to his Honorary Fellowship of the Royal Society of Edinburgh in 1949) as each of his two previous visits had precipitated a major European catastrophe.

Watson's mathematical publications consist of over 150 papers and three books. His first book, *Complex integration and Cauchy's theorem*, was published in 1914. This gave a rigorous proof of Cauchy's theorem based on an initial chapter on "analysis situs", as topology was then called. It also contained a great variety of methods for the evaluation of definite integrals by the calculus of residues. At the present time this book is much less known and used than the other two textbooks of which Watson was an author.

The first of these, *Modern analysis*, had appeared first in 1902, the sole author being the late Sir Edmund Whittaker; it rapidly made a name for itself. The first half contained an account of the methods and processes of analysis, and these were applied, in the second half, to obtain the principal properties of the special functions used in mathematics and its applications. Watson was a friend and former pupil of Whittaker, and he offered to share the work of preparation of a second edition. This proved to be a considerably expanded version of the original work, but followed the same general plan. New chapters on Riemann integration, integral equations and the Riemann zeta-function were added by Watson, and the existing chapters were rewritten to a very considerable extent. For example, the Landau-Pringsheim O, o notation was introduced, so that approximation by asymptotic series and other methods was made more precise. The new edition was an improvement on the old both in the rigour of the treatment and the comprehensiveness of the results included, and has proved its value over the years. Nevertheless, these superior merits were at first not universally acknowledged, and some mathematicians continued to prefer the original edition with its less formal style and greater motivation of general results. In the third edition a further chapter on ellipsoidal harmonics and Lamé's equation was added. At the present time it is probably true to say that the work is still extensively consulted by mathematicians and scientists, not as an exposition of the principles of analysis, but as a compendium of useful results about special functions of all kinds. Possibly Watson was conscious of the fact that the first half of the book was proving less adequate than it had seemed in 1915, since, in his retirement, he embarked on an extensive revision and expansion of the text. At the time of his death he had sketched out the contents of about fifteen chapters on various topics amounting to about 600 quarto manuscript pages. The titles of the first five numbered chapters are: I Introduction; II Natural numbers, numerals and prime numbers; III Fractions and integers; IV Real numbers (including irrational numbers); and V Complex numbers and higher complex numbers. The remaining unnumbered chapters cover a wide field of different topics such as interpolation, asymptotic expansions, linear differential equations, Riemann integration, analytic functions, expansion of functions in infinite series, Jordan's and Cauchy's theorems, inequalities, cyclotomy, etc. Gaps were left to be filled in later, and the incomplete state of the manuscript rules out the possibility of publication. Characteristically, the most complete parts consist of the historical footnotes, which reveal Watson's extraordinary and accurate knowledge of the earlier literature. It may be added that, apart from a chapter on automorphic functions, which he had hoped that Whittaker would write, the entire task of preparing the new version was undertaken by Watson himself. He intended to devote a considerable amount of time to the construction of the real number system. I remember his telling me that he had independently discovered how to describe an ordered pair (x, y) without making such unmathematical statements as that the first member x of the pair is the one on the left.

Much of Watson's work on special functions was concerned with asymptotic expansions. In his long paper **[14]** he developed a general theory of asymptotic series of the type

$$f(x) = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \dots + \frac{a_n}{x^n} + R_n, \tag{1}$$

where, for certain quantities A, B, k, l, ρ, σ independent of n and x,

$$|a_n| \leqslant A\Gamma(kn+1)\rho^n, \quad |R_n x^{n+1}| \leqslant B\Gamma(ln+1)\sigma^n.$$

The formula (1) was assumed to hold when $|x| > \gamma$ and $\alpha \leqslant \arg x \leqslant \beta$. The numbers A, B, k, l, ρ, σ he called the *characteristics* of the expansion (1); they can be regarded as bearing much the same relation to an asymptotic series as the radius of convergence bears to a power series. He investigated under what conditions an asymptotic series for $\Phi\big(f(x)\big)$ can be obtained from (1), where Φ is a suitably well-behaved function, and also to what extent the function f is uniquely determined by the asymptotic series (1). In subsequent papers he applied his powerful analytic methods to a great variety of different special functions. What is known as Watson's Lemma appeared as a lemma in a paper on the parabolic cylinder functions **[38]**. This result enables one to obtain the asymptotic expansion of a function that can be expressed as a Laplace integral, and takes the following form:

Let $\phi(t)$ be an analytic function of t, regular apart from a branch point at 0, when $|t| \leqslant R+\delta$, $|\arg t| \leqslant \Delta < \pi$, where R, δ, Δ are positive; and let

$$\phi(t) = \sum_{m=1}^{\infty} a_m t^{(m/r)-1},$$

when $|t| \leqslant R$, r being positive. Also let $|\phi(t)| \leqslant Ke^{bt}$, where K and b are positive numbers independent of t, when t is positive and $t \geqslant R$. Then

$$\int_0^{\infty} e^{-zt}\phi(t)\,dt \sim \sum_{m=1}^{\infty} a_m \Gamma(m/r) z^{-m/r}$$

as $|z| \to \infty$ in the sector $|\arg z| \leqslant \tfrac{1}{2}\pi - \epsilon < \tfrac{1}{2}\pi$.

The numerous papers on special functions written by Watson between 1910 and 1920 prepared the way for his monumental *Treatise on the theory of Bessel functions*. This work, which extends to more than 800 pages, superseded all earlier books on the subject, and, in a subject where many differing notations had been used, Watson's notations became standard.

The book contains not only formulae and theoretical investigations, but also extensive tables, some of which Watson had himself calculated. Throughout his life he found relaxation in numerical work, for which he used a Brunsviga calculating machine. His first published set of tables [**24**] was offered by him to the British Association Committee on the Calculation of Mathematical Tables, of which he became a member.

During the 1939-1945 war Watson's book on Bessel functions was in great demand in government scientific establishments, both in this country and abroad. It became difficult to acquire and unofficial copies were circulated in some quarters. It was no doubt largely for this reason that a second edition appeared in 1944. By that time, however, Watson had lost interest in the subject and only minimal alterations were made. It is, of course, understandable that his interests should have changed, and a great amount of further work would have had to be done in order to incorporate new work published since the first edition. Nevertheless, as a textbook on Bessel functions the volume is still unsurpassed and is likely to remain so for many years.

During the years between 1928 and the beginning of the Second World War much of Watson's work was concerned with problems connected with the famous Indian mathematician Srinivasa Ramanujan, who died in 1920, and this was Watson's most prolific period. He had received from G. H. Hardy copies of Ramanujan's famous note books (now published in facsimile by the Tata Institute, Bombay) and, like Hardy, he was fascinated by the plethora of formulae of different kinds which followed one after another with scarcely any indication of proof. In a lecture [**71**] to this Society he gives a description of the note books and reveals that he and the late Professor B. M. Wilson, of University College, Dundee, were collaborating in the task of editing them in a form suitable for publication. For this purpose they made a preliminary division of the material, Wilson taking the earlier chapters and Watson the later ones. In a series of numbered papers under the general title "Theorems stated by Ramanujan" both authors published proofs of results taken from the note books. Watson's work is contained in three manuscript files consisting of copies in his very neat hand of the formulae stated by Ramanujan, together with the proofs supplied by Watson himself. These cover Chapters XVI-XXI of Ramanujan's second note book. Gaps were left for proofs to be supplied later on, and these gaps are more frequent in the later chapters. Wilson died in March 1935 and his manuscripts were passed to Watson. The work involved in editing the note books is very considerable and it is scarcely surprising that Watson, probably in the late 1930's, ceased work on it. Through the generosity of Mrs. Watson all this material has been donated to the library of Trinity College, Cambridge, where it will be available to any mathematician who may attempt to complete their work.

Some account of Watson's work on problems associated with Ramanujan is now given.

Watson wrote six long papers on singular moduli [**73**, **74**, **95**, **98**, **103**, **104**]. In the third volume of his *Algebra*, H. Weber develops the theory and gives examples, and other mathematicians such as R. Russell had worked extensively on this subject round about 1890. In his note books Ramanujan constructed 102 singular moduli. Of these 46 had not been given previously by Weber, and Ramanujan published these new results in the *Quarterly Journal of Pure and Applied Mathematics*, 45 (1914), 350–72. Watson's work on singular moduli supersedes that of all other writers on the subject. In all, the number of such moduli that he constructed was over 250, the great majority being new. To indicate the kind of analysis involved, the following description is given. Write $q = e^{\pi i\tau}$ $(\operatorname{Im}\tau > 0)$ and put

$$f_2(\tau) = 2^{\frac{1}{2}} q^{1/12} \prod_{m=1}^{\infty} (1-q^{4m-2})^{-1} = \left(\frac{4k^2}{k'}\right)^{1/12},$$

where k, k' are the usual moduli of elliptic functions having τ as ratio of periods, and let

$$j(\tau) = 256\frac{(1-k^2+k^4)^3}{k^4(1-k^2)^2}$$

be the Dedekind–Klein invariant. Then

$$j(\tau) = \frac{\{f_2^{24}(\tau)+16\}^3}{f_2^{24}(\tau)}$$

and so $-f_2^{24}(\tau)$ is one of the three roots of the equation

$$(x-16)^3 - xj(\tau) = 0.$$

Let n be a positive integer. For purposes of illustration we assume that $n \equiv 3 \pmod 8$. Let h be the class-number of quadratic forms $ax^2+bxy+cy^2$ of determinant $b^2-4ac = -n$. Let $a_r x^2 + b_r xy + c_r y^2$ $(r = 1, 2, \ldots, h)$ be a complete set of such forms and let τ_r be the root with positive imaginary part of $a_r\tau^2 + b_r\tau + c_r = 0$. Then the equation

$$\prod_{r=1}^{h} \{x - j(\tau_r)\} = 0$$

has integral coefficients. Hence so has the equation

$$\prod_{r=1}^{h} \{(x-16)^3 - xj(\tau_r)\} = 0.$$

The latter equation has $3h$ roots, h of which are the numbers $-f_2^{24}(\tau_r)$ $(1 \leqslant r \leqslant h)$. Since we may take τ_1 to be the root of the equation $\tau^2 - \tau + \frac{1}{4}(n-3) = 0$, one of these roots is $-f_2^{24}(\frac{1}{2}+\frac{1}{2}i\sqrt{n}) = F_n^{24}$. These roots can be obtained as radicals. Watson's object was to find the irreducible equation of degree h satisfied by F_n. For this purpose he first reduced the equation of degree $3h$ to one of degree h satisfied by ${F_n}^{24}$, by observing that the $2h$ roots he wished to discard could be paired off into pairs x_1, x_2 satisfying $x_1 x_2 = 2^{12}$. From numerical tables (to 10 decimal places) of the $3h$ roots these pairs could be readily recognised. Each of the remaining h roots is a 24th power x^{24}, say. The selection of the correct 24th root was performed in four stages by finding successively the equations satisfied by x^8, x^4, x^2 and x. To do this the fact that the equation has integral coefficients was used to determine it uniquely. The number of possibilities requiring examination for each n is roughly $3^{\frac{1}{2}(h-1)} + 3 \cdot 2^{\frac{1}{2}(h-1)}$ but many of these can be ruled out quite quickly. Even so, an indication of Watson's accuracy and speed is given by his remark that, for h in the region of 19, about three hours work was required.

Watson was greatly interested [**92, 116**] in problems concerned with congruence properties of various arithmetical functions such as the partition function $p(n)$, defined by

$$\sum_{n=0}^{\infty} p(n) x^n = \prod_{\nu=1}^{\infty} (1 - x^\nu)^{-1} \quad (|x| < 1).$$

He also published tables of some of these functions [**105, 135**]. Ramanujan had conjectured that, if

$$\delta = 5^\alpha 7^\beta 11^\gamma \quad (\alpha, \beta, \gamma = 0, 1, 2, \ldots),$$

then $p(n) \equiv 0 \pmod{\delta}$ whenever $24n \equiv 1 \pmod{\delta}$. This conjecture was known to be false in certain cases and Watson set himself [**116**] the task of finding and proving correct congruences of this type. For this purpose he used his unsurpassed knowledge of theta functions, q-series and modular equations to prove, for example, that $p(n) \equiv 0 \pmod{5^\alpha}$ whenever $24n \equiv 1 \pmod{5^\alpha}$ and $p(n) \equiv 0 \pmod{7^{[(\beta+2)/2]}}$ whenever $24n \equiv 1 \pmod{7^\beta}$. The complication of the proof is considerable and a single formula sometimes runs to ten lines of the wide pages of *Crelle's Journal*. From a remark in [**116**] it looks as if Watson had obtained results also for the modulus 11, but, if so, he did not publish them. A. O. L. Atkin has recently shown that a corresponding result holds for the prime 11, namely $p(n) \equiv 0 \pmod{11^\gamma}$ whenever $24n \equiv 1 \pmod{11^\gamma}$, and has also shown that Watson's proofs for the primes 5 and 7 can be considerably simplified.

Early in 1920, three months before his death, Ramanujan wrote his last letter to Hardy describing his discovery of functions, which he called mock theta functions, and stating some of their properties. The actual definition of what he meant by such a function was not made very precise. Roughly speaking, f is a mock theta function if f has a power series expansion in powers of $q = e^{\pi i \tau}$ for $|q| < 1$ and if we can calculate asymptotic formulae for f as q tends to a "rational point" $e^{2\pi i r/s}$ on the unit circle, of the same degree of precision as can be done for ordinary theta functions. Further, such a function must not be an ordinary theta function. For example, if

$$f(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(1+q)^2(1+q^2)^2\dots(1+q^n)^2},$$

then $f(q)-(-1)^r(1-q)(1-q^3)(1-q^5)\dots(1-2q+2q^4-2q^9+\dots)=O(1)$ at all points $q^r=-1$, but $f(q)=O(1)$ where $q^r=1$.

In his two papers [**97**, **102**] Watson added to Ramanujan's list of mock theta functions and gave proofs of their properties. This he did by obtaining new definitions of the functions by transforming the series by which they are defined into series more amenable to manipulation. For this purpose he used a general formula connecting basic hypergeometric series, which he obtained in a paper [**60**] on the Rogers Ramanujan identities. The first paper [**97**] formed his valedictory address as President of this Society. Watson was always felicitous in his choice of titles (see, for example, [**77**, **136**]); for this paper he made use of the title used by his fictitious namesake in what he imagined was his final memoir on Sherlock Holmes.

Watson's name is also associated with general transforms. The formula

$$f(x)=\int_0^{\infty}\varpi(xu)\,du\int_0^{\infty}\varpi(uy)f(y)\,dy \tag{2}$$

holds under certain conditions for various choices of the kernel function ϖ; thus $\varpi(x)=(2/\pi)^{\frac{1}{2}}\cos x$ gives Fourier's cosine formula, and there are several other examples of kernels ϖ of special forms. If

$$\Omega(s)=\int_0^{\infty}x^{s-1}\varpi(x)\,dx,$$

then it can be shown that, under certain conditions,

$$\Omega(s)\,\Omega(1-s)=1. \tag{3}$$

In his long paper [**78**] Watson considered, among other matters, the problem of obtaining a kind of converse of this. For this purpose he considered any function $\Omega(\frac{1}{2}+it)$ satisfying (3) for which

$$\varpi_1(x)=\frac{x}{2\pi}\underset{T\to\infty}{\text{l.i.m.}}\int_{-T}^{T}\frac{\Omega(\frac{1}{2}+it)}{\frac{1}{2}-it}x^{-\frac{1}{2}-it}\,dt \tag{4}$$

exists. In particular if $\Omega(\frac{1}{2}-it)=\overline{\Omega(\frac{1}{2}+it)}$ and $\Omega(\frac{1}{2}+it)/(\frac{1}{2}-it)$ belongs to $L^2(-\infty,\infty)$ then $\varpi_1(x)$, as defined by (4), exists and $\varpi_1(x)/x\in L^2(0,\infty)$. Let $f\in L^2(0,\infty)$. Then he showed that the formula

$$g(x)=\frac{d}{dx}\int_0^{\infty}\varpi_1(xu)f(u)\frac{du}{u}$$

defines almost everywhere a function $g \in L^2(0, \infty)$ and that the reciprocal formula

$$f(x) = \frac{d}{dx}\int_0^\infty \varpi_1(xu)\, g(u)\frac{du}{u}$$

also holds almost everywhere. Moreover

$$\int_0^\infty \{f(x)\}^2\,dx = \int_0^\infty \{g(x)\}^2\,dx.$$

The connexion between these results and the problem as first stated lies in the fact that essentially

$$\varpi_1(x) = \int_0^x \varpi(u)\,du,$$

so that, if we could carry out the differentiations under the integral signs, we should be able to deduce (2).

Watson's last major paper was on periodic sigma functions [**138**]. In the theory of elliptic functions Weierstrass's sigma function $\sigma(z)$ plays an important ancillary role. Any elliptic function can be expressed as a quotient of sigma functions, but $\sigma(z)$ is not itself an elliptic function. Watson asked the question whether, although not doubly periodic, $\sigma(z)$ could be singly periodic and gave a complete solution to this problem. He found that there existed an enumerable set A of points of the upper half-plane such that if $2\omega_1$ and $2\omega_3$ are periods with quotient $\omega_3/\omega_1 \in A$, then the sigma function formed from these periods is periodic. By suitable choice of ω_1 and ω_3 the fundamental period may be taken to be $4\omega_1$. As usual, Watson combined theoretical and numerical analysis and performed extensive calculations of certain members of the set A and related quantities.

In personal appearance Watson was tall and spare. He had a shy, courtly and slightly formal manner. To his friends, and other acquaintances whom he liked, he could be charming, but, if he disliked someone, he did not hide his feelings. Except in the last few years of his retirement, he kept remarkably fit although he took very little exercise. One of my first impressions of him is of his bounding up the two long flights of stairs to his room in the university, leaving me panting behind.

The mathematics department in Edgbaston consisted of a large room, which housed the lecturing staff, and a smaller room beside it for the professor. Both rooms looked out on the semicircle dominated by the Chamberlain clock tower. Watson was supposed to have discovered a crack on one side of the tower, and it was said that he always passed it at a safe distance on the other side and had calculated that, if it fell, it would not hit his room. He regarded the telephone as an abomination, and he himself did all the secretarial work needed to run his department. This included typing all the examination papers before sending them to the press. It was well known in mathematical circles that letters addressed to him were likely to receive no response and that manuscripts sent to him had little chance of being recovered. This was, however, only partially true. If Watson was interested he would reply at once. As he himself said with characteristic understatement: "I usually reply by return; if not, it may take a little longer." Most of his letters were typed, with formulae and signature written in pencil, but he occasionally wrote by hand in ink and his writing was always very clear. All his letters, on whatever subject, were full of characteristic touches.

One of his great interests, which remained with him all his life, was railways, their early history and their time-tables. It is said that more than one colleague (in a pre-Beeching age) was able to shorten a long journey by making use of Watson's detailed knowledge of minor railway lines and their connexions. As a boy at St. Paul's he chose C. E. Stretton's *History of the Midland Railway* as his First Prize in the Mathematical 8th Form, and the fortunes of this company remained his particular interest throughout his life. A manuscript of about a dozen chapters entitled *Three decades of Midland Railway locomotives* was left uncompleted at his death. This monograph was to have been submitted to the *Journal of the Stephenson Locomotive Society* and its subtitle reads "A compilation of the locomotives owned by the company during the period when Mathew Kirtley was locomotive superintendent, *i.e.* May 1844–May 1873". Only five of the chapters are numbered and their titles are of some interest: I Introduction, II Archaeography, III More Archaeography, VI The Dark Ages, VII Bulk Buying. In his retirement his interests alternated between his projected revision of *Modern analysis* and this monograph. In this connexion, some indication of his style as a letter writer is given by the following extract from a letter to the writer in July 1962: ". . . since the middle of January I have deserted Urania (I think that she was the Muse who took charge of Mathematics as a side-line) in favour of Clio, and the latter is proving a most exacting Mistress. All my time and energy have been spent in coping with her. I spent until about the middle of April in attacking the problems in which I was interested (you will not be surprised at hearing that they involve lots and lots of numbers—all integers) . . . ." These numbers were, of course, locomotive numbers. Another life-long hobby was stamp collecting, and he possessed an enormous collection arranged in more than thirty boxes.

After he retired in 1951 Watson insisted on withdrawing his entry from *Who's Who*, but retained his wide interests in mathematics and other subjects. Every Friday during term time he used to come up to Edgbaston from his home in Leamington Spa and spend a few hours in the university visiting friends and tearing up old examination scripts for scrap paper.

He had a remarkable memory for factual information of every kind, including the dates of birth of all his acquaintances.

In July 1961 the new mathematics building at Birmingham University was opened and christened the Watson Building. A plaque commemorating Watson's association with the university was unveiled at the same time. When the proposal to name the building after him was broached, Watson was at first not very keen, but after consideration he consented and it became clear that he derived considerable pleasure from this gesture. With his passing we have lost a most interesting personality and one of the great mathematicians of the classical school of analysis.

Publications

Books

Complex integration and Cauchy's theorem, Cambridge Tracts in Mathematics and Mathematical Physics, No. 15, Cambridge University Press, 1914.

A course of modern analysis (with E. T. Whittaker), Cambridge University Press, 2nd Ed. 1915, 3rd Ed. 1920, 4th Ed. 1927 (Reprinted 1935, 1940, 1946, 1950, 1952, 1958, 1962).

A treatise on the theory of Bessel functions, Cambridge University Press, 1st Ed. 1922, 2nd Ed. 1944 (Reprinted 1952, 1958, 1962, 1966).

Papers

1. "A method of determining a very rapidly convergent series for the square root of any positive integer", *Math. Gaz.*, 2 (1903), 361.
2. "The general solution of Laplace's equation in n dimensions", *Mess. Math.*, 36 (1906), 98–106.
3. "The expansions of products of hypergeometric functions", *Quart. J. Pure Appl. Math.*, 39 (1907), 27–51.
4. "A series for the square of the hypergeometric function", *Quart. J. Pure Appl. Math.*, 40 (1908), 46–57.
5. "A note on the solution of the linear difference equation of the first order", *Quart. J. Pure Appl. Math.*, 41 (1909), 10–20.
6. "On a certain difference equation of the second order", *Quart. J. Pure Appl. Math.*, 41 (1909), 50–55.
7. "The cubic transformation of the hypergeometric function", *Quart. J. Pure Appl. Math.*, 41 (1909), 70–79.
8. "The solution of the homogeneous linear difference equation of the second order", *Proc. London Math. Soc.* (2), 8 (1910), 125–161.
9. "The solution of a certain transcendental equation", *Proc. London Math. Soc.* (2), 8 (1910), 162–177.
10. "The continuations of functions defined by generalized hypergeometric series", *Trans. Cambridge Phil. Soc.*, 21 (1910), 281–299.
11. "The harmonic functions associated with the parabolic cylinder," *Proc. London Math. Soc.* (2), 8 (1910), 393–421.
12. "The singularities of functions defined by Taylor's series", *Quart. J. Pure Appl. Math.*, 42 (1910), 41–53.
13. "A note on equipotential curves", *Mess. Math.*, 40 (1911), 152–160.
14. "A theory of asymptotic series", *Phil. Trans. Roy. Soc.* (A), 211 (1911), 279–313.
15. "The characteristics of asymptotic series", *Quart. J. Pure Appl. Math.*, 43 (1911), 63–77.
16. "The solution of the homogeneous linear difference equation of the second order", *Proc. London Math. Soc.* (2), 10 (1911), 211–248.

17. "The transformation of an asymptotic series into a convergent series of inverse factorials", *Rend. Circ. Mat. Palermo*, 34 (1912), 41–88.
18. "Some solutions of Laplace's equation", *Proc. London Math. Soc.* (2), 12 (1913), viii–xii.
19. "Some analytic functions associated with the *G*-function", *Quart. J. Pure Appl. Math.*, 44 (1913), 187–205.
20. "A class of integral functions defined by Taylor's series", *Trans. Cambridge Phil. Soc.*, 22 (1913), 15–37.
21. "Some properties of the extended zeta function", *Proc. London Math. Soc.* (2), 12 (1913), 288–296.
22. "The convergence of the series in Mathieu's functions", *Proc. Edinburgh Math. Soc.*, 33 (1915), 25–30.
23. "An inequality associated with the gamma function", *Mess. Math.*, 45 (1915), 28–30.
24. "Tables of the logarithmic gamma function, its integral and derivative", *British Association Annual Reports*, 1916, 123–126. See also *British Association Mathematical Tables*, 1 (1931), xvii and Table X.
25. "A problem of analysis situs", *Proc. London Math. Soc.* (2), 15 (1916), 227–242.
26. "The sum of a series of cosecants", *Phil. Mag.* (6), 31 (1916), 111–118.
27. "Bessel functions of equal order and argument", *Phil. Mag.* (6), 32 (1916), 232–237.
28. "A note on meromorphic functions", *Quart. J. Pure Appl. Math.*, 47 (1917), 302–310.
29. "Some simple expansion formulae", *Mess. Math.*, 46 (1917), 97–101.
30. "Bessel functions of equal order and argument", *Proc. Cambridge Phil. Soc.*, 19 (1917), 42–48.
31. "The limits of applicability of the principle of stationary phase", *Proc. Cambridge Phil. Soc.*, 19 (1917), 49–55.
32. "Bessel functions and Kapteyn series", *Proc. London Math. Soc.* (2), 16 (1917), 150–174.
33. "Simple types of Kapteyn series", *Mess. Math.*, 46 (1917), 150–157.
34. "Bessel functions of large order", *Proc. Cambridge Phil. Soc.*, 19 (1917), 96–110.
35. (with G. A. Schott) "Asymptotic formulae occurring in electron theory", *Quart. J. Pure Appl. Math.*, 47 (1915), 311–333.
36. "A theorem of Liouville's type", *Mess. Math.*, 47 (1917), 107–113.
37. "An expansion related to Stirling's formula, derived by the method of steepest descents", *Quart. J. Pure Appl. Math.*, 48 (1917), 1–18.
38. "The harmonic functions associated with the parabolic cylinder" (Second paper), *Proc. London Math. Soc.* (2), 17 (1918), 116–148.
39. "The zeros of Bessel functions", *Proc. Roy. Soc.* (A), 94 (1918), 190–206.
40. "The integral formula for generalized Legendre functions", *Proc. London Math. Soc.* (2), 17 (1918), 241–246.
41. "Approximate formulae for Legendre functions", *Mess. Math.*, 47 (1918), 151–160.
42. "On Nielsen's functional equations", *Mess. Math.*, 48 (1918), 49–53.
43. "Bessel functions of equal order and argument", *Phil. Mag.* (6), 35 (1918), 364–370.
44. "The diffraction of electric waves by the earth", *Proc. Roy. Soc.* (A), 95 (1918), 83–99.
45. "The problem of the square pyramid", *Mess. Math.*, 48 (1918), 1–22.
46. "Asymptotic expansions of hypergeometric functions", *Trans. Cambridge Phil. Soc.*, 22 (1918), 277–308.
47. "The transmission of electric waves round the earth", *Proc. Roy. Soc.* (A), 95 (1919), 546–563.
48. "The zeros of Lommel's polynomials", *Proc. London Math. Soc.* (2), 19 (1920), 266–272.
49. "The product of two hypergeometric functions", *Proc. London Math. Soc.* (2), 20 (1921), 189–195.
50. "The theorems of Clausen and Cayley on products of hypergeometric functions", *Proc. London Math. Soc.* (2), 22 (1923), 163–170.
51. "Dixon's theorem on generalized hypergeometric functions", *Proc. London Math. Soc.* (2), 22 (1923), i–iii (meeting of May 17).

52. " The sum of a series of cosecants ", *Phil. Mag.* (6), 45 (1923), 577–581.

53. " A note on generalized hypergeometric series ", *Proc. London Math. Soc.* (2), 22 (1923), iii–v (meeting of November 8); also in 23 (1924), xiii–xv.

54. " Trinity College in the time of Newton "; from *Isaac Newton* 1642–1727 (edited by W. J. Greenstreet for the Mathematical Association), London 1927, pp. 144–147.

55. " Some integrals involving Bessel functions ", *Journal London Math. Soc.*, 3 (1928), 22–27.

56. " An expansion due to Abel ", *Journal London Math. Soc.*, 3 (1928), 188–192.

57. " Theorems stated by Ramanujan II: Theorems on summation of series ", *Journal London Math. Soc.*, 3 (1928), 216–225.

58. " Theorems stated by Ramanujan IV: Theorems on approximate integration and summation of series ", *Journal London Math. Soc.*, 3 (1928), 282–289.

59. " Question 722 " (S. Ramanujan), *J. Indian Math. Soc.*, 18 (1929), 113–117.

60. " A new proof of the Rogers Ramanujan identities ", *Journal London Math. Soc.*, 4 (1929), 4–9.

61. " Theorems stated by Ramanujan V: Approximations connected with e^x ", *Proc. London Math. Soc.* (2), 29 (1929), 293–308.

62. " Theorems stated by Ramanujan VII: Theorems on continued fractions ", *Journal London Math. Soc.*, 4 (1929), 39–48.

63. " Theorems stated by Ramanujan VIII: Theorems on divergent series ", *Journal London Math. Soc.*, 4 (1929), 82–86.

64. " Theorems stated by Ramanujan IX: Two continued fractions ", *Journal London Math. Soc.*, 4 (1929), 231–237.

65. " The use of series of Bessel functions in problems connected with cylindrical wind-tunnels ", *Proc. Roy. Soc.* (A), 130 (1930), 29–37.

66. " The constants of Landau and Lebesgue ", *Quart. J. of Math.* (Oxford), 1 (1930), 310–318.

67. (with T. Vijayaraghavan) " Question 784 (S. Ramanujan) ", *J. Indian Math. Soc.*, 19 (1931), 12–23.

68. " Theorems stated by Ramanujan XI ", *Journal London Math. Soc.*, 6 (1931), 59–65.

69. " Theorems stated by Ramanujan XII: A singular modulus ", *Journal London Math. Soc.*, 6 (1931), 65–70.

70. " Theorems stated by Ramanujan XIV: A singular modulus ", *Journal London Math. Soc.*, 6 (1931), 126–132.

71. " Ramanujan's note books ", *Journal London Math. Soc.*, 6 (1931), 137–153.

72. " Some self-reciprocal functions ", *Quart. J. of Math.* (Oxford), 2 (1931), 298–309.

73. " Some singular moduli (I) ", *Quart. J. of Math.* (Oxford), 3 (1932), 81–98.

74. " Some singular moduli (II) ", *Quart. J. of Math.* (Oxford), 3 (1932), 189–212.

75. " The congress at Zürich ", *Math. Gaz.*, 16 (1932), 297–300.

76. " Über die Schläflischen Modulargleichungen ", *Vorhandlungen des Internationales Mathematiker Kongresses Zürich*, 1932, Vol. II, p. 13 (Zürich, 1933).

77. " The marquis and the land agent; a tale of the eighteenth century ", *Math. Gaz.*, 17 (1933), 5–17.

78. " General transforms ", *Proc. London Math. Soc.* (2), 35 (1933), 156–199.

79. " Proof of certain identities in combinatory analysis ", *J. Indian Math. Soc.*, 20 (1933), 57–69.

80. " Du Bois Reymond's constants ", *Quart. J. of Math.* (Oxford), 4 (1933), 140–146.

81. " Über die Schläflischen Modulargleichungen ", *J. Reine Angew. Math.*, 169 (1933), 238–251.

82. " Notes on generating functions of polynomials : (1) Laguerre polynomials ", *Journal London Math. Soc.*, 8 (1933), 189–192.

83. " Notes on generating functions of polynomials : (2) Hermite polynomials ", *Journal London Math. Soc.*, 8 (1933), 194–199.

84. " Notes on generating functions of polynomials : (3) Polynomials of Legendre and Gegenbauer ", *Journal London Math. Soc.*, 8 (1933), 289–292.

85. " Generating functions of class-numbers ", *Compositio Math.*, 1 (1934), 39–68.

86. " Scraps from some mathematical note-books ", *Math. Gaz.*, 18 (1934), 5–18.
87. " Notes on generating functions of polynomials : (4) Jacobi polynomials ", *Journal London Math. Soc.*, 9 (1934), 22–28.
88. " An infinite integral involving Bessel functions ", *Journal London Math. Soc.*, 9 (1934), 16–22.
89. " An inequality ", *Quart. J. of Math.* (Oxford), 5 (1934), 221–223.
90. " A problem of distribution ", *Math. Gaz.*, 18 (1934), 245–247.
91. " Ramanujan's continued fraction ", *Proc. Cambridge Phil. Soc.*, 31 (1935), 7–17.
92. " Über Ramanujansche Kongruenzeigenschaften der Zerfällungsanzahlen I ", *Math. Zeitschrift*, 39 (1935), 712–731.
93. " Über Poissonsche Summationsformeln ", *Comptes-Rendus 3ème Congres des Math. des pays Slaves, Prague*, 1934 (1935), 169–171.
94. " Über eine Formel für numerische Berechnung der bestimmten Integrale ", *Časopis Pest. Mat. Fys.*, 65 (1935), 1–7.
95. " Singular moduli (3) ", *Proc. London Math. Soc.* (2), 40 (1935), 83–142.
96. " The surface of an ellipsoid ", *Quart. J. of Math.* (Oxford), 6 (1935), 280–287.
97. " The final problem : an account of the mock theta functions ", *Journal London Math. Soc.*, 11 (1936), 55–80.
98. " Singular moduli (4) ", *Acta Arith.*, 1 (1936), 284–323.
99. " Ramanujan's integrals and Gauss's sums ", *Quart. J. of Math.* (Oxford), 7 (1936), 175–183.
100. " A note on parabolic cylinder functions ", *Journal London Math. Soc.*, 11 (1936), 250–251.
101. " An integral equation for the square of a Laguerre polynomial ", *Journal London Math. Soc.*, 11 (1936), 256–261.
102. " The mock theta functions (2) ", *Proc. London Math. Soc.* (2), 42 (1936), 274–304.
103. " Singular moduli (5) ", *Proc. London Math. Soc.* (2), 42 (1937), 377–397.
104. " Singular moduli (6) ", *Proc. London Math. Soc.* (2), 42 (1937), 398–409.
105. " Two tables of partitions ", *Proc. London Math. Soc.* (2), 42 (1937), 550–556.
106. " A note on Spence's logarithmic transcendant ", *Quart. J. of Math.* (Oxford), 8 (1937), 39–42.
107. " A note on Lerch's functions ", *Quart. J. of Math.* (Oxford), 8 (1937), 43–47.
108. " Comments on Note 1225 ", *Math. Gaz.*, 21 (1937), 292–295.
109. " A series identity ", *Edinburgh Math. Notes*, 30 (1937), iii–iv.
110. " Studio di una particolare funzione definita da un integrale improprio ", *Rend. Accad. Naz. dei Lincei* (6), 26 (1937), 91–96.
111. " Über eine Differentialgleichung des Fuchsschen Typus ", *Math. Annalen*, 115 (1938), 237–248.
112. " A note on the polynomials of Hermite and Laguerre ", *Journal London Math. Soc.*, 13 (1938), 29–32.
113. " Some integrals involving Bessel functions ", *Journal London Math. Soc.*, 13 (1938), 41–44.
114. " Two infinite integrals ", *Proc. Edinburgh Math. Soc.* (2), 5 (1938), 174–181.
115. " Tabellazione di una particolare funzione definita da un integrale improprio ", *Rend. Accad. Naz. dei Lincei* (6), 27 (1938), 525–528.
116. " Ramanujan's Vermutung über Zerfällungsanzahlen ", *J. Reine Angew. Math.*, 179 (1938), 97–128.
117. " A note on Gegenbauer polynomials ", *Quart. J. of Math.* (Oxford), 9 (1938), 128–140.
118. " Über eine Reihe aus verallgemeinerten Laguerre'schen Polynomen ", *S.-B. Akad. Wiss. Wien Math.-naturw. Kl.* (IIa), 147 (1938), 151–159.
119. " A note on the polynomials of Hermite and Laguerre ", *Journal London Math. Soc.*, 13 (1938), 29–32.
120. " Cubic envelopes derived from systems of conics ", *Math. Gaz.*, 22 (1938), 261–264.
121. " On Brun's modification of Simpson's rule ", *Norske Vid. Selsk. Skr. Trondheim* No. 4 (1938), 1–14.
122. " Über Eigenschaften der Ramanujanschen Kettenbruches ", *Monatsh. Math. Phys.*, 48 (1939), 516–530.

123. " A postscript to Maunsell's problem in cartophily ", *Math. Gaz.*, 23 (1939), 78–79.
124. " Another note on Laguerre polynomials ", *Journal London Math. Soc.*, 14 (1939), 19–22.
125. " Three triple integrals ", *Quart. J. of Math.* (Oxford), 10 (1939), 266–276.
126. " A logical point in dynamics ", *Math. Gaz.*, 23 (1939), 475–476.
127. " An inequality in the triangle ", *Quart. J. of Math.* (Oxford), 11 (1940), 273–276.
128. " Proof of a conjecture stated by Mordell ", *Journal London Math. Soc.*, 16 (1941), 157–166.
129. " An analytical proof of Pascal's theorem ", *Math. Gaz.*, 25 (1941), 109–110.
130. " An infinite integral ", *Proc. Cambridge Phil. Soc.*, 38 (1942), 323–324.
131. " On a functional equation ", *Math. Gaz.*, 28 (1944), 218–220.
132. " Pascal's theorem again ", *Math. Gaz.*, 28 (1944), 220–222.
133. " A problem in elementary analysis proposed by Mordell ", *Proc. London Math. Soc.* (2), 48 (1945), 391–400.
134. " Two Tripos questions ", *Math. Gaz.*, 29 (1945), 221–224.
135. " A table of Ramanujan's function $\tau(n)$ ", *Proc. London Math. Soc.* (2), 51 (1948), 1–13.
136. " A comedy of errors ", *Math. Gaz.*, 32 (1948), 15–16.
137. " A curious property of the prime number 503 ", *Math. Gaz.*, 33 (1949), 302–303.
138. " Periodic sigma functions ", *Proc. London Math. Soc.* (3), 2 (1952), 129–149.
139. " Two inequalities ", *Math. Gaz.*, 37 (1953), 244–246.
140. " A reduction formula ", *Proc. Glasgow Math. Assoc.*, 2 (1954), 57–61.
141. " Schur's inequality ", *Math. Gaz.*, 39 (1955), 207–208.
142. " Two more Tripos questions ", *Math. Gaz.*, 39 (1955), 280–286.
143. " A pretty series ", *Math. Gaz.*, 39 (1955), 297.
144. " A note on convergence ", *Math. Gaz.*, 39 (1955), 297–299.
145. " Some identities associated with a discriminant ", *Proc. Edinburgh Math. Soc.* (2), 10 (1956), 101–107.
146. " A trivial inequality ", *Math. Gaz.*, 40 (1956), 288.
147. " Summations by calculation of probabilities ", *Math. Gaz.*, 40 (1956), 291–293.
148. " A bilinear transformation ", *Edinburgh Math. Notes*, 40 (1956), 1–7.
149. " Comments on ' A note on infinite series ' ", *Math. Gaz.*, 41 (1957), 51–52.
150. " Another pretty series ", *Math. Gaz.*, 41 (1957), 219–220.
151. " A theorem on continued fractions ", *Proc. Edinburgh Math. Soc.* (2), 11 (1959), 167–174.
152. " A note on gamma functions ", *Edinburgh Math. Notes*, 41 (1959), 7–9.
153. " A quadruple integral ", *Math. Gaz.*, 43 (1959), 280–283.
154. " A proof of Kirkman's hypothesis ", *Proc. Edinburgh Math. Soc.*, 13 (1962), 131–138.

Department of Mathematics,
The University,
Glasgow, W.2.