and of intellectual honesty. Those who were privileged to be his pupils will always cherish the memory of his kindliness and his splendid sincerity.

Fry published the following papers in the *Proceedings of the Royal Irish Academy*:—

"The centre of gravity and the principal axes of any surface of equal pressure in a heterogeneous liquid covering a heterogeneous solid composed of nearly spherical shells of equal density when the whole mass is rotating with a small angular velocity in relative equilibrium under its own attraction". (Vol. 27 A, 139.)

"Real and complex numbers considered as adjectives or operators". (Vol. 32 A, 15.)

"Impact in three dimensions". (Vol. 33 A, 75.)

He also edited the second edition of Burnside and Panton's *Theory of Equations*.

JOHN RAYMOND WILTON

H. S. Carslaw *and* G. H. Hardy.

John Raymond Wilton, Professor of Mathematics in the University of Adelaide, South Australia, died on 12 April, 1944.

He was born on 4 May, 1884. His father was on the literary staff of an Adelaide newspaper. He entered the University early in 1901 and obtained his B.Sc. at the end of 1903, with first class honours in both mathematics and physics. Prof. Bragg, later Sir William, then held the combined chairs of Mathematics and Physics, and he once described him as having had the greatest natural genius for mathematics among any of his students during his more than twenty years in Adelaide. On his advice Wilton proceeded to Cambridge, entering Trinity in October 1904. In March 1905 he was awarded a sizarship, later converted into a major scholarship. In the Tripos of 1907 he was fifth wrangler, the first five places being filled by Trinity men: G. N. Watson was senior, H. W. Turnbull second, and A. V. Hill third. Even in so good a year Wilton might have gained a higher place.

In the following year, on the advice of his private tutor Herman, he took Part II of the Natural Science Tripos, gaining a first class in physics. The next twelve months he worked in the Cavendish, did some lecturing in mathematics to students of physics, and some demonstrating. There can be no doubt that in forsaking mathematics for natural science he made a serious mistake. In spite of his first class he was not interested in physics, and he was not an applied mathematician. He would have attained distinction as an analyst much sooner had he taken Part II of the Mathematical Tripos in 1908 and devoted the next year to advanced study in mathematics.

In October 1909 he was appointed Assistant Lecturer in Mathematics at Sheffield. From 1912 to 1916 he published many papers in the *Messenger of Mathematics*, *Philosophical Magazine* and *Quarterly Journal of Mathematics*. They were mainly concerned with partial differential equations, hydrodynamics and elliptic functions. His first contributions to the *Proceedings* of this Society were his papers **22** and **23**. In 1916 (17) he had already turned his attention to the ζ-function of Riemann.

From 1916 to 1922 he published nothing. The war affected him very deeply, and his connection with Sheffield ended in 1916. He had been doing some X-ray work in a hospital there before conscription was introduced, and he hoped to be allowed to continue it; but the tribunal before which he appeared directed him to St. George's Hospital, London, where for two years, till the end of the war, he was kept copying "cases" from one book into another. His financial resources were small and both he and his wife suffered many privations. He was teaching in the Friends School at Saffron Walden for a short time after the war and about this time he became a member of the Society of Friends.

In 1919 he was appointed to a Lectureship in Mathematics in Manchester under Horace Lamb, and at the end of that year he was offered the chair at Adelaide of which Lamb had been the first holder, during 1876–1885.

When Bragg left Adelaide in 1908, a Professor of Physics was appointed, but mathematics, regarded as the handmaiden of engineering and physics, was placed under the supervision of the Professor of Engineering. This arrangement, which we may assume to have been intended to be temporary, lasted till a year after the end of the war. In 1919 it was decided that a Professor of Mathematics should be appointed, and the position was offered to Wilton, who arrived in Adelaide in January 1920 to take up his duties. He found the mathematical courses at the University much the same as those which he had attended some twenty years earlier, and for thirty years there had been little change in the mathematics taught in the schools. In South Australia secondary education had not made as much progress as in New South Wales and Victoria. His first task was a complete renovation of the work in mathematics at the University and a revision of the programmes in mathematics for the examinations controlled by the University, which then governed to a great extent the curricula of the schools in the state. Though he was thus able materially to improve the standard of the pass work of the first year, it remained for many years lower than he would have liked.

The pass classes in his department were large in the first year, 80–110, and much smaller in the second and third years, 30–50 and 10–20. His programme for both pass and honours was an ambitious one, stronger naturally in pure mathematics than in applied. In 1923 H. W. Sanders was appointed Lecturer and took over all the applied work. To him Wilton also handed over the large pass class of the first year; and he was thus able to devote himself to the pure mathematics of the second and third years, and to the courses offered to such students as were willing to undertake at least part of the programme for honours. This covered four years, but from 1920 to 1940 only eight persons graduated with honours in mathematics. In Sydney it was not unusual in these years to have more than that number of honours graduates in mathematics annually. Wilton envied N.S.W. its High Schools scattered through the state, the large endowments of its university, and its many graduate scholarships: Adelaide had only the 1851 Exhibition and the Rhodes Scholarships. Of one of its Rhodes Scholars, who promises to be the most distinguished mathematician Australia has produced, he had good reason to be proud.

He visited England only once after his appointment to Adelaide (December 1924–August 1925), but he corresponded with a number of fellow-workers, British and foreign. Among the latter were Landau, Walfisz and Hasse, and by the last named he was asked to contribute to *Crelle's Journal*. He had by now found his proper field, and from 1922 to 1934 he was really productive; but he was attacked by ill-health in 1934 and wrote practically nothing after. He explains this in a letter of 6 June 1941. He had made no attempt, he said, to do anything mathematical for years, the effect of overstrain, from which it had taken him a long time to recover; but in spite of a severe illness three years earlier he felt himself in better health than for years and he had been trying to pick up the threads again. In 1934 he had sent a long paper "On the ζ-function of Riemann", in two parts, to the *Quarterly Journal* and the *Proceedings of the Cambridge Philosophical Society*. Both parts had been accepted but returned to him for revision, owing to his ignorance of Siegel's analysis, in 1931, of Riemann's surviving manuscripts. He had put his paper aside and had not looked at it again till the beginning of 1941; but he had since been working through it and thought he would be able to make something of it when he got into his stride again. Two days later he told his wife that he had "seen something that he had been looking for for years", and that he felt he could now go straight ahead with the paper, which he was re-writing. That evening he had a paralytic stroke, which left him with speech and memory seriously impaired. He made a partial recovery, and after two years began again to do some work at his department, but he was never the same man again. On 9 April 1944 he had a second stroke and he died three days later.

He had taken his D.Sc. in Adelaide in 1912 and he was given a Cambridge Sc.D. in 1930. In 1934 he received the Lyle Medal of the Australian National Research Council, awarded for the best work in mathematics or physics produced in Australia in the five-year period, 1928–1933.

He had become one of the most influential leaders in Australia of the Society of Friends. He was a great lover of Dante and before his illness he was in the habit of reading through the *Divina Commedia* in the original several times a year. He used to say that this work had exercised the greatest single influence on his life. Late in life he discovered a love and deep understanding of classical music.

He was married twice. First, in England in 1910, to Annie Martha Gladstone of Forest Hill, London: she died in 1932 after a long illness. There were no children of this marriage. Second, in 1936, to Winifred Welbourn of Adelaide: she and a daughter born in 1937 survive him.

The most important of Wilton's papers are no doubt those (**30–1**, **33–4**, **36–7**, **46**, **50**) concerning "Gauss's circle problem", "Dirichlet's divisor problem", and their extensions; but the analysis in them is intricate and it is not easy to summarize his chief results. It has been familiar since Gauss that, if $r(n)$ is the number of representations of n by two squares, so that

$$R(x) = \sum_{n \leqslant x} r(n) = \sum_{u^2+v^2 \leqslant x} 1$$

is the number of integral lattice points in the circle $u^2+v^2 \leqslant x$, then $R(x) = \pi x + O(\sqrt{x})$, the area of the circle with an error of the order of the circumference. If we write

$$R(x) = \pi x + 1 + \mathrm{P}(x),$$

then a variety of problems arise concerning $\mathrm{P}(x)$. These are of two kinds, problems of order, and problems of identity, concerning the exact expression of $\mathrm{P}(x)$ as a convergent series of Bessel functions. These form the content of the "circle problem", and the "divisor problem" covers the similar problems for the divisor function $d(n)$ and the rectangular hyperbola $uv \leqslant x$. There are a multitude of generalizations associated with more general figures, such as m-dimensional spheres and ellipsoids, with weighted lattice points, and with functions defined by sums of powers of divisors.

Wilton's work concerns the identities connected with these problems. It is known, for example, that

$$\mathrm{P}(x) = \sqrt{x}\sum_{1}^{\infty}\frac{r(n)}{\sqrt{n}} J_1\{2\pi\sqrt{(nx)}\},$$

the series being convergent; and each generalization of the problem introduces a similar identity, generally with a summable divergent series. The widest generalizations are those of Oppenheim, Walfisz, and Wilton himself. It is hardly possible to state them without elaborate explanations, but there is a good historical account of the subject, up to 1929, in **31**, and this makes Wilton's own contributions clear.

Another arithmetical function which had an invincible attraction for Wilton was Ramanujan's $\tau(n)$. In **38**, for example, he investigates the "identities" connected with this function, and proves that

$$\frac{1}{\Gamma(a+1)}\sum_{n\leqslant x}(x-n)^a\tau(n) = \frac{1}{(2\pi)^a}\sum_{1}^{\infty}\left(\frac{x}{n}\right)^{6+\frac{1}{2}a}\tau(n)\,J_{12+a}\{4\pi\sqrt{(nx)}\}$$

if $a>0$; the result was afterwards extended by Hardy [*PCPS*, 34 (1938)] to cover the case $a=0$. This paper also contains other results of considerable interest; the functional equation

$$(2\pi)^{-s}\Gamma(s)F(s) = (2\pi)^{s-12}\Gamma(12-s)F(12-s)$$

for $F(s)=\Sigma n^{-s}\tau(n)$, and a proof that $F(s)$ has an infinity of zeros on $\Re s=6$ (an analogue of a known theorem concerning Riemann's ζ-function). He was also much interested in the curious congruence properties of $\tau(n)$, and **40** contains what are perhaps the most general results known about them. He did further work in this direction which has never been published, and there is a long manuscript about the coefficients in

$$\{x^{\frac{1}{24}}(1-x)(1-x^2)\ldots\}^q,$$

for general q, still in the possession of the Society: the coefficient is $\tau(n)$ when $q=24$.

A man with these interests was inevitably attracted by Poisson's formula, and Voronoï's more recondite formula for sums $\Sigma d(n)f(n)$. In **47** he extends the work of Dixon and Ferrar on Voronoï's formula, and in **48** he applies the results to series such as

$$\Sigma n^{-a}d(n)\,{\cos \atop \sin}\,2n\pi\theta,$$

already considered by Chowla and Walfisz. His various short notes on Fourier analysis, and Poisson's formula in particular, not of great importance in themselves, may be regarded as studies preliminary to this work.

Finally, another interesting pair of papers is **51–2**, which contain almost his last work. Here he simplifies some intricate analysis of van der Corput and applies it to trigonometrical series. In particular he proves that if ϕ and ψ are sufficiently regular functions such that $\psi' \to \infty$ and $\phi^2 > A\psi''$, where A is constant, then

$$\Sigma\, \phi(n)\, e^{2\pi i(n\theta - \psi(n))}$$

diverges for all θ. If $\psi = n \log n$, $\phi = n^{-\frac{1}{2}}$, we obtain a theorem of Hardy and Littlewood; if $\psi = n \log\log n$, $\phi = (\log n)^{-1}$, one of Steinhaus.

It will be plain from this summary that Wilton was a fine mathematician, with admirable taste and a natural inclination towards deep and difficult problems. He may have left nothing, strictly, of major importance, but his record is genuinely impressive. He might perhaps have made a bigger name if his taste had been less fine, and he had been content to work in fields which offer cheaper rewards.

We have to thank Prof. H. W. Sanders, who has succeeded him at Adelaide, for providing a list of Wilton's published papers and information about the mathematical courses in the University.

List of papers by J. R. Wilton.

[*JLMS*, Journal London Math. Soc.; *PLMS*, Proc. London Math. Soc. (2); *M*, Messenger of Math.; *QJ*, Quarterly Journal of Math.; *PCPS*, Proc. Camb. Phil. Soc.; *PRS*, Proc. Royal Soc. (A); *PM*, Phil. Mag.]

1. Note on the solution of a certain partial differential equation. *M*, 42 (1912).
2. Note on the solution of the equation $r = f(t)$. *M*, 43 (1913).
3. On the solution of an equation of the form $F(r, s, t) = 0$. *M*, 43 (1913).
4. Some simple transformations of Stokes' current function. *M*, 43 (1913).
5. Note on the equation $s = f(z)$. *M*, 43 (1913).
6. On plane waves of sound. *PM* (6), 26 (1913).
7. On the highest wave in deep water. *PM* (6), 26 (1913).
8. On the potential and force function of an electrified spherical bowl. *M*, 44 (1914).
9. Note on a certain partial differential equation of the second order possessing an intermediate integral of the first order. *M*, 44 (1914).
10. A simple transformation of certain partial differential equations. *M*, 44 (1914).
11. The seven-fourteen section of the Zeta function. *M*, 44 (1914).
12. The nine-eighteen section of the Zeta function. *M*, 44 (1914).
13. The eleven-twentytwo section of the Zeta function. *M*, 44 (1914).
14. On deep water waves. *PM* (6), 27 (1914).
15. Figures of equilibrium of rotating fluid under the restriction that the figure is to be a surface of revolution. *PM* (6), 28 (1914).
16. A transformation of the partial differential equation of the second order. *M*, 45 (1915).
17. Note on the zeros of Riemann's ζ-function. *M*, 45 (1915).
18. On ripples. *PM* (6), 29 (1915).
19. On the solution of certain problems of two-dimensional physics. *PM* (6), 30 (1915).

20. On the conditions that certain partial differential equations of the second order have an intermediate integral of the same order. *QJ*, 46 (1915).
21. A continued fraction solution of the linear differential equation of the second order. *QJ*, 46 (1915).
22. On Darboux's method of solution of partial differential equations of the second order. *PLMS*, 14 (1915).
23. A pseudo-sphere whose equation is expressible in terms of elliptic functions. *PLMS*, 14 (1915).
24. A formula in zonal spherical harmonics. *M*, 46 (1916).
25. Certain criteria for the success of Darboux's method when applied to the equation $s = f(x, y, z, p, q)$. *QJ*, 47 (1916).
26. A proof of Burnside's formula for $\log \Gamma(x+1)$ and certain allied properties of Riemann's Zeta function. *M*, 52 (1923).
27. The Gibbs phenomenon in series of the Schlömilch type. *M*, 56 (1927).
28. A note on the coefficients in the expansion of $\zeta(s, z)$ in powers of $(s-1)$. *QJ*, 50 (1927).
29. The approximate functional formula for the Theta function. *JLMS*, 2 (1927).
30. The lattice points of an n-dimensional ellipsoid. *JLMS*, 2 (1927).
31. The lattice points of a circle; an historical account of the problem. *M*, 58 (1928).
32. Some applications of a transformation of series. *PLMS*, 27 (1928).
33. The average value of an exponential function over the lattice points of a circle. *PCPS*, 24 (1928).
34. A series of Bessel functions connected with the lattice points of an n-dimensional ellipsoid. *PRS*, 120 (1928).
35. The Gibbs phenomenon in Fourier-Bessel series. *Journal f. Math.*, 159 (1928).
36. A series of Bessel functions connected with the theory of lattice points. *PLMS*, 29 (1929).
37. The lattice points of a circle. *Proc. Royal Soc. Edinburgh*, 48 (1929).
38. A note on Ramanujan's arithmetical function $\tau(n)$. *PCPS*, 25 (1929).
39. On Ramanujan's arithmetical function $\Sigma_{r,s}(n)$. *PCPS*, 25 (1929).
40. Congruence properties of Ramanujan's function $\tau(n)$. *PLMS*, 31 (1930).
41. An approximate functional equation for the product of two ζ-functions. *PLMS*, 31 (1930).
42. The mean value of the Zeta function on the critical line. *JLMS*, 5 (1930).
43. A proof of Poisson's summation formula. *JLMS*, 5 (1930).
44. Congruence properties of Ramanujan's function $\tau(n)$ to the modulus 11. *Boletin. Mat., Buenos Aires* (June 1930).
45. A proof of Fourier's Theorem. *JLMS*, 6 (1931).
46. On Dirichlet's divisor problem. *PRS*, 134 (1931).
47. Voronoï's summation formula. *QJ* (Oxford), 3 (1932).
48. An approximate functional equation with applications to a problem of Diophantine approximation. *Journal f. Math.*, 169 (1933).
49. A note on Stirling's theorem. *Mathematical Notes* (Edinburgh Math. Soc., No. 28, 1933).
50. An extended form of Dirichlet's divisor problem. *PLMS*, 36 (1934).
51. An approximate functional equation of a simple type (I). *JLMS*, 9 (1934).
52. An approximate functional equation of simple type (II): applications to certain trigonometrical series. *JLMS*, 9 (1934).

Also in South Australia.

Certain Diophantine problems. *Proc. Royal Soc. of South Australia*, 44 (1920).
Notes on the mathematical syllabus of the Public Examinations (1921).