

## LUIGI CREMONA. 1830—1903.

Luigi Cremona, whose death occurred on June 10, 1903, was born at Pavia on December 7, 1830. At the age of 18 he served as a volunteer in the War of Liberation, and shared in the defence of Venice up to its capitulation. Returning to Pavia, he entered the University, where he studied mathematics under Bordoni and Brioschi, in company with Beltrami, Casorati and others. After holding minor appointments at Pavia, Cremona, and Milan successively, he became in 1860 Professor of Higher Geometry at Bologna. In 1866 he returned to Milan as Professor of Geometry and Graphical Statics in the University, and remained there until he was called to Rome in 1873, as University Professor of Higher Mathematics, and Director of the Engineering School, which was re-organised under his supervision. These posts he held until the time of his death. As Senator and Member of the Higher Council of Public Instruction (of which he was several times the head) he was able to influence the development of the national education of Italy; and it was he who secured the introduction of projective geometry and graphical statics into the official programme of studies. His official duties at Rome engrossed his time, and after 1879 he published only a few mathematical papers: three of these were contributed in 1884 to the London Mathematical Society, the Royal Society of Edinburgh, and the Royal Irish Academy, respectively. It was in 1884 that he paid a visit to this country, on the occasion of the centenary celebration of the University of Edinburgh.

Cremona's natural genius for geometry was happily stimulated in various ways; primarily, according to his own statement, by the *Aperçu Historique* and *Géométrie Supérieure* of Chasles. When he had mastered the method of projective geometry, he was exceptionally qualified to show the power of analysis and pure geometry in combination; for his early training in analysis under Brioschi must have been thorough, and his own sympathies became more and more attracted by pure geometry. In this respect it is interesting to compare him with Salmon, who maintained the balance between the two methods with remarkable steadiness. Cremona, about 1860, was about equally interested in both: he remained, perhaps, equally capable, but not equally interested. Thus we find that, in later years, he recognized Staudt as the true founder of pure geometry, whereas at an earlier date Staudt's purism had somewhat repelled him.

The change in attitude is already marked in the celebrated memoir on cubic surfaces which in 1866 secured half the Steiner prize offered by the Academy of Berlin, the other half being awarded to Sturm.

This is called a memoir of pure geometry, and it is so in the sense that no explicit analytical formulæ are used in it. But Cremona clearly saw that the propositions of Steiner which he was asked to prove depended in great measure on properties of surfaces of any degree; so he began with an outline of the general theory of surfaces, *assuming* as known the fundamental properties of polar surfaces. Now it is quite true that polar curves and surfaces can be defined in geometrical terms, but this definition is artificial, and the question of degree (which is essential in most of the applications) cannot be decided, except by algebraical considerations, or by an extension of Staudt's theory of imaginary points which is extremely laborious, and not fruitful in results. Thus we cannot help feeling that the memoir begins by a sort of evasion; and every now and then we suspect the author of having translated into a geometrical form a proof obtained by analysis. Nevertheless, the memoir is a splendid contribution to geometry in the proper sense; for even if analytical proofs were supplied for the propositions most easily proved in that way, it would not weaken the impression produced by the whole: we should still feel that the writer is dealing with geometrical facts, and engaged in geometrical speculation. The same may be said of Cremona's use of the method of enumeration; hazardous as it is in the hands of the incompetent, he employed it with great effect in arriving at geometrical conclusions: leaving to others the task of verifying his results in a more rigorous way. In this he was following the example of Chasles, Cayley, Salmon and others.

Cremona did not again compete for the Steiner prize; but it was awarded to him in 1874 in recognition of his geometrical researches. It was indeed well deserved; for he had then published his treatises on plane curves and on surfaces, as well as most of his papers of first-rate importance. Among these are the researches, most closely associated with his name, on the birational transformations of plane and solid space, as well as of curves and surfaces. The most familiar example of a birational transformation of a plane is that of inversion: this is a particular case of the quadratic transformation which, in its normal form, is—

$$x : y : z = y'z' : z'x' : x'y'$$

leading to

$$x' : y' : z' = yz : zx : xy.$$

The first consideration of this appears to be due to Magnus and Steiner; Cremona extended the method indefinitely by observing that if we put  $x : y : z = \phi : \chi : \psi$ , where  $\phi, \chi, \psi$  are polynomials in  $x', y', z'$  of the  $n$ th degree such that, with  $a, b, c$  constant,  $a\phi + b\chi + c\psi = 0$  goes through  $(n^2 - 1)$  fixed points, there is a one-one correspondence between  $(x', y', z')$  and  $(x, y, z)$ . There is a similar theory for solid

space, but it has not been worked out in detail. Cremona's theory of transformation interested him all his life, and his last published paper (Tr. R. Irish Acad., 1884) is on a transformation of the fourth order in space of three dimensions, the inverse transformation being of the sixth order. His method attracted universal attention, and has proved to be of the highest importance, not only in geometry, but in the analytical theory of algebraic functions and integrals. It is, in fact, an algebraic theory, though it is convenient to state it in a geometrical form, and its author preferred to regard it from a geometrical point of view.

Nearly connected with the foregoing is the representation of surfaces on a plane. Map-drawing is a familiar example, which engaged the attention of Hooke and possibly Mercator, as well as of Lagrange, Gauss, &c. For a one-one correspondence the surface must be unicursal, and this is sufficient; Cremona is associated with Cayley, Clebsch, Nöther and others, in the development of the theory. He also applied it to cubic surfaces, Steiner's surface, various scrolls, and some singular surfaces with cuspidal curves.

Among his minor papers may be noticed those on twisted cubics, unicursal twisted quartics, and the three-cusp hypocycloid. Like all his work they are written in the most clear and attractive style: in fact, he may be compared with Dirichlet in his power of simplifying and illuminating everything on which he wrote, even when the subject was not altogether new.

Mathematical students all over the world are indebted to Cremona for his truly admirable treatises: *Introduzione ad una teoria delle curve piane* ("Mem. Acc. Bologna," 1862); *Preliminari ad una teoria geometrica delle superficie* (ibid., 1866); *Elementi di Geometria proiettiva* (Rome, 1873); *Elementi di calcolo grafico* (Turin, 1874); and *Le figure reciproche della statica grafica* (3rd ed., Milan, 1879). These, and the translations of them into foreign languages, must have greatly helped the progress of the subjects with which they deal; they are lucid, elegant, and stimulating, and are not surpassed in merit by any text-books of their kind.

Cremona was a member of all the leading academies of the world, and received, in May, 1903, from the German Emperor, the order *Pour le mérite*. He was elected Foreign Member of the Royal Society in 1879, but did not make any communications to the "Transactions" or "Proceedings."

Obituary notices of some length by D'Ovidio and Veronese have appeared in the *Rendiconti* of the Academy of Turin and of the *Accademia dei Lincei*. From the second of these assistance has been derived in the preparation of the present notice; it concludes with a list of Cremona's scientific publications.

G. B. M.