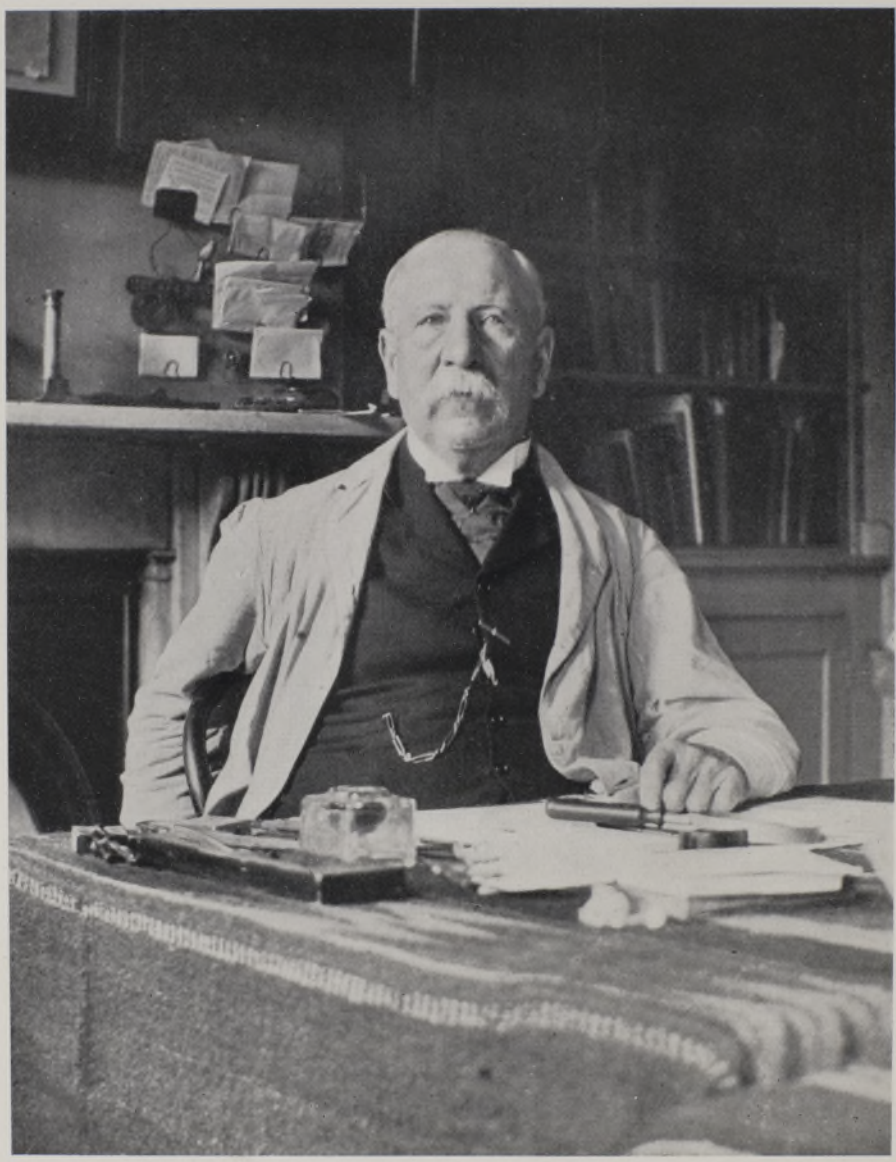

A. G. Greenhill

# ALFRED GEORGE GREENHILL, 1847–1927.

Alfred George Greenhill, born November 29, 1847, elected F.R.S. in 1888, died February 10, 1927. After a distinguished career at Christ's Hospital, and at St. John's College, Cambridge, he was second wrangler in 1870, but bracketed with the Senior Wrangler, Richard Pendlebury, of his own college, in the Smith Prize Examination. In the same year he was elected Fellow of the College. Shortly afterwards he was appointed to the Royal Indian Engineering College, Cooper's Hill. In 1873, however, he was made Fellow and Lecturer at Emmanuel College, Cambridge. This he left in 1876 to become Professor of Mathematics to the Advanced Class of Artillery Officers at Woolwich. After more than thirty years' government service he retired, and the rest of his life was spent, as a bachelor, in London, first in New Inn, and later in Staple Inn; in his later years, he was the recipient of a Civil List Pension. During his residence in London he was very active in mathematical and scientific circles; he became a Fellow of the Royal Society in 1888, and received a Royal Medal from that Society in 1906; he was awarded the De Morgan Medal by the London Mathematical Society in 1902; he had served as President of this Society 1890–92. He was on the Councils of both societies, of the Royal Society, in 1896, 1897, and of the London Mathematical Society for many years. He also became a corresponding member of the Académie des Sciences (Paris), and a foreign member of the Accademia dei Lincei (Rome). He was knighted in 1908, on his retirement from Woolwich.

His original contributions to knowledge were mainly to Dynamics, to Hydrodynamics, and to Elasticity. Of the great value of these a highly appreciative account is given by an expert, in Prof. Love's Obituary Notice of Greenhill, "Journal of the London Math. Soc.," vol. iii, pp. 29 and 30, 1928. Prof. Love writes as follows:—

"Greenhill made valuable contributions to hydrodynamics. In the theory of the motion of a solid in a fluid, developed by Kelvin, Kirchhoff, and others, the fluid is regarded as acting only by pressure and inertia, and obeying the electrical law of flow. Greenhill applied this theory to give an account of the steadiness of flight conferred upon an elongated projectile by rifling. He determined the least angular velocity about its axis for which steady motion of a solid of revolution, moving in the direction of its axis, can be stable. This was done incidentally in a paper on 'Fluid motion between confocal ellipsoids and confocal elliptic cylinders' ('Quart J. of Math.,' vol. 16 (1879), p. 227). The ballistic implications of the result, in regard to the degree of rifling required for various types of projectiles, were pointed out in his article 'Hydromechanics' in the 10th edition of the 'Encyclopædia Britannica,' where it is stated that

they agree fairly well with what has been found in practice. Probably there was some earlier publication of these results. This practical application of what was regarded as a recondite mathematical theory earned for him much renown at Woolwich. Some years later he took up the theory which traces fluid resistance to discontinuity, and wrote a long ' Report on stream line motion past a plane barrier ' (1910) for the Advisory Committee on Aeronautics. In this he gave an account of the theory, as previously developed, and extended it by completing the solutions of various special problems that require elliptic functions. At that time progress was just beginning to be made with the difficult problem of curved boundaries, and he wrote afterwards a supplementary report (1916), dealing with the researches in this department up to date, and completing many of them in the same way. But his contributions to hydrodynamics do not begin and end with applications of elliptic functions. The rifling problem does not involve them, though the paper in which it was solved introduces them, and they play no part in the investigation that he made of the problem, first attacked by Lejeune Dirichlet, of the motion of a liquid ellipsoid, subject to its own attraction, and moving so as to remain ellipsoidal. Riemann had shown that in all such motions the co-ordinates of a fluid particle must be linear functions of their initial values, and had obtained the equations which the coefficients of these linear functions must satisfy. Greenhill showed that all the possible motions in which the ellipsoid retains its form can be obtained by compounding irrotational motion of liquid in a rotating ellipsoidal shell with vortex motion of the liquid rotating as if rigid, thus interpreting Riemann's result for such motions, and arriving at it by a more intuitive method (' Proc. Camb. Phil. Soc.,' vol. 4 (1883), p. 4).

" Perhaps the most striking of Greenhill's contributions to the theory of elasticity is his paper on ' . . . . height consistent with stability . . . . ' (' Proc. Camb. Phil. Soc.,' vol. 4 (1883), p. 65). Here he attacked for the first time the question of the greatest length which a shaft standing upright can have if it is not to be bent by its own weight. In Euler's well-known strut problem the weight of the shaft is neglected, and the tendency to bend arises from a supported load. The problem turned out to be easy, and Greenhill made applications of the result to the question of the greatest height to which a tree can grow. As always, he took great pains to ascertain the observed facts, and, in this case, he collected interesting data in regard to the heights of exceptional trees, comparing them with his result. Another important contribution to the theory of elastic stability was his investigation (' Proc. Lond. Inst. Mech. Engineers ' (1883), p. 182) of the condition for the whirling of shafts, in regard to which he brought out the importance of the gravest period of lateral vibration. He made out that a long shaft, rotating between bearings, can remain straight if its rotation period exceeds this gravest vibrational period. If it rotates too fast it may whirl. Reference has already

been made to his elaborate discussion of the problem of the deformation of an elastic circular ring by pressure ('Math. Ann.,' vol. 52 (1899), p. 465). At the time it was hoped that the solution of this rather ideal problem would throw light on the baffling, but technically important, problem of the collapse of boiler flues, but the solution of this was to be found much later, along quite different lines, and by another hand."

But Greenhill's contributions to the mathematical and technical journals were many and voluminous. A large part of many of his papers is taken up with detailed computations with elliptic functions. One can respect his desire to present the solution of his problems in finished form, and can understand his enthusiasm when one reflects that he was largely a pioneer in making known the applications of these functions; but work of this kind is apt to miss appreciation on both sides. To the physicist it means the necessity of understanding much detailed algebra, and is less convincing than an approximate solution by elementary functions; to the mathematician its special character and lack of horizon, all that makes it suitable finally for numerical estimates, renders it extremely cramping and dull. Moreover, in Greenhill's time there were no adequate numerical tables of the elliptic functions; it is understood that it is due largely to his efforts that the Smithsonian Institution undertook to supply this want, and so placed the scientific world under great obligation.

To judge of his mathematical achievements one must in fairness recall the outlook of the English Universities in mathematics at a time, say, about forty years ago. It is clear that to his contemporaries of that time he was an advanced thinker. But with tastes which were apparently primarily mathematical he was impatient of systematic theory, and desired always to be in touch with some practical application, without being willing to give himself to prolonged experimental work. To many in the practical world he stood, it would seem, as the apostle of higher mathematical methods; to the mathematicians in England, however, whose theoretical outlook was constantly being enlarged, he was persistent in his denunciation of much that had emerged since his own student years, unless capable of deduction by the algebraic methods in which he was so great an adept.

His characteristics are indelibly printed on the text books he wrote, all of them arresting, and full of points of novelty—books that have a permanent value as milestones in our pædagogy of mathematics in England, but each requiring special study of its method of arrangement. His second, on the applications of Elliptic Functions, 1892, which was translated into French, was a pioneer work: it develops the addition theorem in connection with the motion of a pendulum, and explains Weierstrass' factors for the sigma function in connection with the plane flow of electricity. This is very interesting to anyone, and seems admirable from the point of view from which the book was written;

but it is worse than inadequate in its mutilation of a wonderful theory, painfully won over many years by many workers, by more germane methods. His first book, on the 'Differential and Integral Calculus,' 1886, is smaller, but also practical, especially in its introduction of integration concurrently with differentiation. His most valuable book, probably, on 'Hydrostatics,' 1894, is enriched by the introduction of many of the actual problems of the science, and marked a great advance on the material which had been called by the same name for many years. All these books have most carefully drawn diagrams, costing the author infinite trouble.

But, apart from Greenhill's scientific discoveries, and the influence of his educational outlook, there is more to say, and that the best which can be said of any man. His personality, apart from a certain wilful pose of eccentricity (*cf.* 'The Times Obituary,' 14 February, 1927), was, especially to younger men, and in the earlier years of his professorial activities, inspiring by its earnestness in a manner not easy to put into words; his kindliness, and interest in the work of others, made a great impression on his friends; he will be greatly missed.

H. F. B.

The portrait, here reproduced, of Sir George Greenhill was taken in August, 1916, in his study at Staple Inn, by Mr. J. W. Hicks, M.B.E., F.R.A.S., who kindly lent the photograph.