DAVID HILBERT

1862-1943

David Hilbert, upon whom the world looked during the last decades as the greatest of the living mathematicians, died in Gottingen, Germany, on 14 February 1943. At the age of eighty-one he succumbed to a compound fracture of the thigh brought about by a domestic accident.

Hilbert was born on 23 January 1862, in the city of Konigsberg in East Prussia. He was descended from a family which had long been settled there and had brought forth a series of physicians and judges. During his entire life he preserved uncorrupted the Baltic accent of his home. For a long time Hilbert remained faithfully attached to the town of his forbears, and well deserved its honorary citizenship which was bestowed upon him in his later years. It was at the University of Konigsberg that he studied, where in 1884 he received his doctor's degree, and where in 1886 he was admitted as Privatdozent; there, moreover, he was appointed Ausserordentlicher Professor, in 1892, succeeding his teacher and friend Adolf Hurwitz, and in the following year advanced to a full professorship. The continuity of this Konigsberg period was interrupted only by a semester's studies at Erlangen, and by a travelling scholarship during the year before his habilitation, which took him to Felix Klein at Leipzig and to Paris where he was attracted mainly to Ch. Hermite. It was on Klein's initiative that Hilbert was called to Gottingen in 1895; there he remained until the end of his life. He retired in the year 1930.

In 1928 he was elected to foreign membership of the Royal Society.

Beginning in his student years at Konigsberg a close friendship existed between him and Hermann Minkowski, his junior by two years, and it was with deep satisfaction that in 1902 he succeeded in bringing Minkowski also to Gottingen. Only too soon did the close collaboration of the two friends end with Minkowski's death in 1909. Hilbert and Minkowski were the real heroes of the great and brilliant period, unforgettable to those who lived through it, which mathematics experienced during the first decade of this century in Gottingen. Klein ruled over it like a distant god, 'divus Felix', from above the douds; the peak of his mathematical productivity lay behind him. Among the luthors of the great number of valuable dissertations which in these fruitful fears were written under Hilbert's guidance we find many Anglo-Saxon names, lames of men who subsequently have played a considerable role in the development of American mathematics. The physical set-up within which this free scientific life unfolded was quite modest. Not until many years after the first world war, after Felix Klein had gone and Richard Courant had succeeded him, owards the end of the brief period of the German Republic, did Klein's dream of the Mathematical Institute at Gottingen come true. But soon the Nazi storm broke and those who had laid the plans and who taught there with Hilbert were scattered over the earth, and the years after 1933 became for him tragic years of ever deepening loneliness.

Hilbert was of slight build. Above the small lower face with its goatee there rose the dome of a powerful, in later years bald, skull. He was physically agile, a tireless walker, a good skater, and a passionate gardener. Until 1925 he enjoyed good health. Then he fell ill of pernicious anaemia. Yet this illness only temporarily paralyzed his restless activity in teaching and research. He was among the first with whom the liver treatment, inaugurated by G. R. Minot at Harvard, proved successful; undoubtedly it saved Hilbert's life at that time.

Hilbert's research left an indelible imprint on practically all branches of mathematical science. Yet in distinct successive periods he devoted himself with impassioned exclusiveness to but a single subject at a time. Perhaps his deepest investigations are those on the theory of number fields. His monumental report on the 'Theorie der algebraischen Zahlkorper', which he submitted to the Deutsche Mathematiker-Vereinigung, is dated as of the year 1897, and as far as I know Hilbert did not publish another paper in this field after 1899. The methodical unity of mathematics was for him a matter of belief and experience. It appeared to him essential that—in the face of the manifold interrelations and for the sake of the fertility of research—the productive mathematician should make himself at home in all fields. To quote his own words: 'The question is forced upon us whether mathematics is once to face what other sciences have long ago experienced, namely to fall apart into subdivisions whose representatives are hardly able to understand each other and whose connexions for this reason will become ever looser. I neither believe nor wish this to happen; the science of mathematics as I see it is an indivisible whole, an organism whose ability to survive rests on the connexion between its parts.' Theoretical physics also was drawn by Hilbert into the domain of his research; during a whole decade beginning in 1912 it stood at the centre of his interest. Great, fruitful problems appear to him as the life nerve of mathematics. 'Just as every human enterprise prosecutes final aims', says he, 'so mathematical research needs problems. Their solution steels the force of the investigator.' In his famous lecture at the International Congress of Mathematicians at Paris in 1900 Hilbert tries to probe the immediate future of mathematics by posing twenty-three unsolved problems; they have indeed, as we can state to-day in retrospect, played an eminent role in the development of mathematics during the subsequent forty-three years. A characteristic feature of Hilbert's method is a peculiarly direct attack on problems, unfettered by any algorithms; he always goes back to the questions in their original simplicity. When it is a matter of transferring the theory of linear equations from a finite to an infinite number of unknowns he begins by getting rid of the calculatory tool of determinants. A truly great example of far reaching significance is his mastery of Dirichlet's principle which, originally springing from mathematical physics, provided Riemann with the foundation of his theory of algebraic functions and Abelian integrals, but which subsequently fell victim to Weierstrass's pitiless criticism. Hilbert salvaged it in its entirety. The whole finely wrought apparatus of the Calculus of Variations was consciously set aside here. We only need mention the names R. Courant and M. Morse to indicate what role this direct method of the Calculus of Variations was destined to play in recent times. It seems to me that with Hilbert, the mastering of individual concrete problems and the formation of general abstract concepts are balanced in a particularly fortunate manner. He came from a time in which the algorithm had played a more extensive role, and therefore he strongly emphasized a conceptual procedure; but in the meantime, our progress in this direction has been so uninhibited and with so little concern for a growth of the problems in depth that many of us have begun to fear for the mathematical substance. In Hilbert, simplicity and rigor go hand in hand. The growing demand for rigor, imposed by the critical reflections of the nineteenth century upon those parts of mathematics that operate in the continuum, was felt by most investigators as a heavy yoke that made their steps dragging and awkward. Full of longing and with uneasiness, they looked back upon Euler's era of happy-go-lucky analysis. With Hilbert, rigor no longer appears as an enemy, but as a promoter of simplicity. Yet the secret of Hilbert's creative force is not plumbed by any of these remarks. A further element of it, I feel, was his sensitivity in registering hints, which revealed to him general relations while solving specific problems. This is most magnificently exemplified by the way in which, during his theory of numbers, he was led to the enunciation of his general theorems on class fields and the general law of reciprocity.

In a few words we shall now recall Hilbert's most important achievements. In the years 1888-1892 he proved the fundamental finiteness theorems of the *theory of invariants* for the full projective group. His method, though yielding proof for the existence of a finite basis for the invariants, does not actually enable one to construct it in a concrete individual case. Hence the exclamation by the great algorithmicist P. Gordan, at the appearance of Hilbert's paper: "This is not mathematics; this is theology!" It reveals an antithesis which reaches down to the very roots of mathematics. Hilbert, however, in further penetrating investigations, furnished the means for a finite execution of the construction

His papers on the theory of invariants had the unexpected effect of withering, as it were overnight, a discipline which until then had stood in full bloom. Its central problems he had finished once and for all. Entirely different was his effect on the *theory of number fields*, which he took up in the years 1892-1898. It is a great pleasure to watch how, step by step, in a succession of papers ascending from the particular to the general, the appropriate concepts and methods are evolved and the essential connections come to light. These papers proved of extraordinary fertility for the future. On the pure theory of numbers side, I mention the names of Furtwängler, Takagi, Artin, Hasse, and Chevalley, and on the number-and-function theoretical side, those of Fuëter and Hecke.

During the subsequent period, 1898-1902, the foundations of geometry were closest to Hilbert's heart, and he was seized by the idea of axiomatics. The soil was well prepared, especially by the Italian school of geometers. Yet it was as if over a landscape, wherein but a few men with a superb sense of orientation had found their way in murky twilight, the sun had risen all at once. Clear and clean-cut we find stated the axiomatic concept according to which geometry is a hypothetical deductive system; it depends on the ‘implicit definitions’ of the concepts of spatial objects and relations which the axioms contain, and not on a description of their intuitive content. A complete and natural system of geometric axioms is set up. They are required to satisfy the logical demands of consistency, independence, and completeness, and by means of quite a few peculiar geometries, constructed *ad hoc*, the proof of independencThe general ideas appear to us to-day almost banal, but in these examples Hilbert unfolds his typical wealth of invention. While in this fashion the geometric concepts become formalized, the logical ones function as before in their intuitive significance. The further step where logic too succumbs to formalization, thus giving rise to a purely symbolic mathematics—a step upon which Hilbert already pondered at this epoch, as a paper read to the International Congress of 1904 proves, and which is inevitable for the ultimate justification of the role played by the infinite in mathematics—was systematically followed up by Hilbert during the final years of his mathematical productivity, from 1922 on. In contrast to L. E. J. Brouwer's intuitionism, which finds itself forced to abandon major parts of historical mathematics as untenable, Hilbert attempts to save the holdings of mathematics in their entirety by proving its formalism free of contradiction. Admittedly the question of truth is thus shifted into the question of consistency. To a limited extent the latter has been established by Hilbert himself in collaboration with P. Bernays, by J. von Neumann, and G. Gentzen. In recent times, however, the entire enterprise has become questionable on account of K. Godel's surprising discoveries. While Brouwer has made clear to us to what extent the intuitively certain falls short of the mathematically provable, Godel shows conversely to what extent the intuitively certain goes beyond what (in an arbitrary but fixed formalism) is capable of mathematical proof. The question of the ultimate foundations and the ultimate meaning of mathematics remains open; we do not know in which direction it will find its final solution nor even whether a final objective answer can be expected at all. 'Mathematizing' may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization.

A chance occasion, a lecture in 1901 by the Swedish mathematician E. Holmgren in Hilbert's seminar dealing with the now classical paper of Fredholm's on *integral equations*, then but recently published, provided the impulse which started Hilbert on his investigations on this subject which absorbed his attention until 1912. Fredholm had limited himself to setting up the analogue of the theory of linear equations, while Hilbert recognized that the analogue of the transformation on to principal axes of quadratic forms yields the theory of the eigenvalues and eigenfunctions for the vibration problems of physics. He developed the parallel between integral equations and sum equations

in infinitely many unknowns, and subsequently proceeded to push ahead from the spectral theory of 'completely continuous' to the much more general one of 'bounded' quadratic forms. To-day these things present themselves to us in the framework of a general theory of Hilbert space. Astonishing indeed is the variety of interesting applications which integral equations find in the most diverse branches of mathematics and physics. Mention should be made of Hilbert's own solution of Riemann's problem of monodromy for linear differential equations, a far reaching generalization of the existence theorem for algebraic functions on a preassigned Riemann surface, and his treatment of the kinetic theory of gases, also the completeness theorem for the representations of a continuous compact group, and finally in recent times the construction of harmonic integrals on an arbitrary Riemannian manifold, successfully accomplished by the use of Hilbertian means. Thus only under Hilbert's hands did the full fertility of Fredholm's great idea unfold. But it was also due to his influence that the theory of integral equations became a world-wide fad in mathematics for a considerable length of time, producing an enormous literature, for the most part of rather ephemeral value. It was not merit but a favour of fortune when, beginning in 1923 (Heisenberg, Schrodinger) the spectral theory in Hilbert space was discovered to be the adequate mathematical instrument of quantum physics. This later impulse led to a re-examination of the entire complex of problems with refined means (J. von Neumann, M. , and others).

This period of integral equations is followed by Hilbert's physical period. Significant though it was for Hilbert's complete personality as a scientist, it produced a lesser harvest than the purely mathematical phases, and may here be passed over. I shall mention instead two single, somewhat isolated, accomplishments that were to have a great effect: his vindication of Dirichlet's principle; and his proof of a famous century-old conjecture of Waring's, carrying the statement that every integer can be written as a sum of four squares over from squares to arbitrary powers. The physical period is finally succeeded by the last one, already mentioned above, in the course of which Hilbert gives an entirely new turn to the question concerning the foundation and the truth content of mathematics itself. A fruit of Hilbert's pedagogic activity during this period is the charming book by him and Cohn-Vossen, *Anschauliche Geometrie*.

This summary, though far from being complete, may suffice to indicate the universality and depth of Hilbert's mathematical work. He has impressed the seal of his spirit upon a whole era of mathematics. And yet I do not believe that his research work alone accounts for the brilliance that eradiated from him, nor for his tremendous influence. Gauss and Riemann, to mention two other Gottingers, were greater mathematicians than Hilbert, and yet their immediate effect upon their contemporaries was undoubtedly smaller. Part of this is certainly due to the changing conditions of time, but the character of the men is probably more decisive. Hilbert's was a nature filled with the zest of living, seeking the intercourse of other people, and delighting in the exchange of scientific ideas. He had his own free manner of learning and teaching. His comprehensive mathematical knowledge he acquired not so much from lectures as in conversations with Minkowski and Hurwitz. 'On innumerable walks, at times undertaken day by day', he tells in his obituary on Hurwitz, 'we browsed in the course of eight years through every corner of mathematical science.' And as he had learned from Hurwitz, so he taught in later years his own pupils—on far-flung walks through the woods surrounding Gottingen or, on rainy days, as peripatetics, in his covered garden walk. His optimism, his spiritual passion, and his unshakable faith in the value of science were irresistibly infectious. He says: 'The conviction of the solvability of each and every mathematical problem spurs us on during our work; we hear within ourselves the steady call: there is the problem; search for the solution. You can find it by sheer thinking, for in mathematics there is no ignorabmus.' His enthusiasm didcriticism, but not with scepticism. The snobbish attitude of pretended indifference, of 'merely fooling around with things' or even of playful cynicism, did not exist in his circle. Hilbert was enormously industrious; he liked to quote Lichtenberg's saying: 'Genius is industry'. Yet for all this there was light and laughter around him. Under the influence of his dominating power of suggestion one readily considered important whatever he did; his vision and experience inspired confidence in the fruitfulness of the hints he dropped. It is moreover decisive that he was not merely a scientist but a scientific personality, and therefore capable not only of teaching the technique of his science but also of being a spiritual leader. Although not committing himself to one of the established epistemological or metaphysical doctrines, he was a philosopher in that he was concerned with the life of the idea as it realizes itself among men and as an indivisible whole; he had the force to evoke it, he felt responsible for it in his own sphere, and measured his individual scientific efforts against it. Last, not least, the environment also helped. A university such as Gottingen, in the halcyon days before 1914, was particularly favourable for the development of a living scientific school. Once a band of disciples had gathered around Hilbert, intent upon research and little worried by the toil of teaching, it was but natural that in joint competitive aspiration of related aims each should stimulate the other; there was no need that everything come from the master. His homeland and America among all countries felt Hibert's impact most thoroughly. His influence upon American mathematics was not restricted to his immediate pupils. Thus, for instance, the Hilbert of the foundations of geometry had a profound effect on E. H. Moore and O. Veblen; the Hilbert of integral equations on George D. Birkhoff.

A picture of Hilbert's personality should also touch upon his attitude regarding the great powers in the lives of men; social and political organization, art, religion, morals and manners, family, friendship, love. Suffice it to say here that he was singularly free from all national and racial prejudices, that in all questions, political, social, or spiritual, he stood forever on the side of freedom, frequently in isolated opposition to the compact majority of his environment. Unforgotten by all those present remains the unanimous and prolonged applause which greeted him in 1928 at Bologna, the first International Congress of Mathematicians at which, following a lengthy struggle, the Germans were once more admitted. It was a telling expression of veneration for the great mathematician whom every one knew to have risen from a severe illness, but at the same time an expression of respect for the independent attitude, 'au dessus de la melee', from which he had not wavered during the world conflict. With veneration, gratitude and love his memory will be preserved beyond the gates of death by many a mathematician throughout the world.

Hermann Weyl