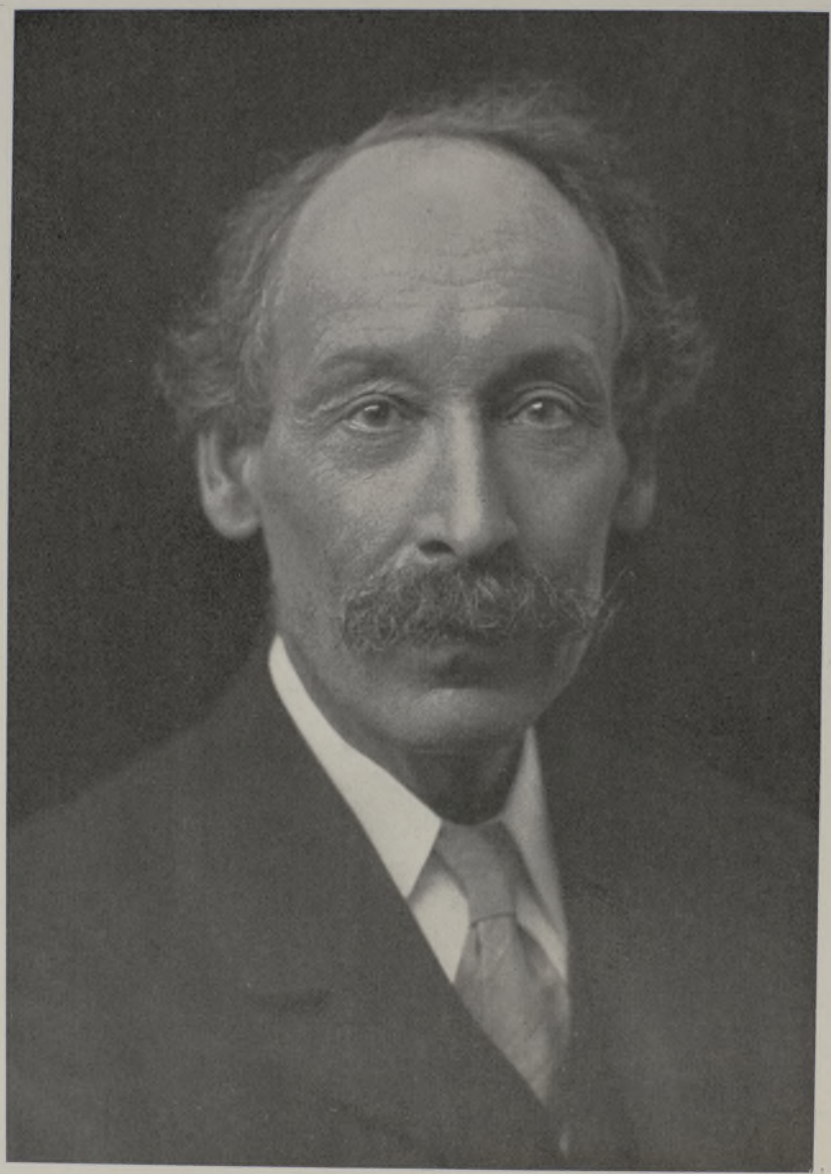

M. J. M. Hill

MICAIAH JOHN MULLER HILL, 1856–1929.

Micaiah John Muller Hill was the eldest son of the Rev. Samuel John Hill and was born at Berhampore, Bengal, on February 22, 1856, during the stormy days of the Indian Mutiny, in which he narrowly escaped death. He was educated at the School for the Sons of Missionaries, Blackheath, and entered University College, London, as a student in October, 1872. His academic career in London was one long sequence of brilliant successes and at the same time an arduous struggle against financial difficulties which he could overcome only by winning such scholarships as the College and the University of London had to offer in those days—rare and coveted distinctions which fell to the lot of very few.

In 1874 he took his B.A. degree in London, obtaining the first place in the Mathematical Honours List, a feat he accomplished in only two years and which he followed up in 1876 by winning the Gold Medal in the M.A. Examination. By this time he had entered as an undergraduate at Peterhouse, Cambridge. In 1879 he was Fourth Wrangler and Smith's Prizeman. He then returned to University College for a few months as assistant in the Department of Mathematics, and, in 1880, at the age of twenty-four, was elected to the Chair of Mathematics at Mason College, as it then was, now the University of Birmingham.

It was during his tenure of the Chair at Birmingham that Hill produced his earliest research work, which dates from 1883. In that year he published three papers, one on a generalisation of Tesseral Harmonics for more than two variables, another on the equation giving the anharmonic ratios of the roots of a quintic, and the third, which started an important group in Hill's earlier work, on a generalisation of the equations of Hydrodynamics for space of n dimensions. This was immediately followed by a paper developing the theory of cylindrical vortices of finite section, moving in an infinite fluid, in particular of the elliptic cylindrical vortex. This paper is notable as being Hill's first contribution to the Transactions of the Society.

In 1884 Hill was called from Birmingham to fill the Chair of Mathematics at University College, London, in succession to R. C. Rowe. His colleagues at the time included T. G. Bonney, Carey Foster, Charles Graham, L. F. Vernon Harcourt, Victor Horsley, Alexander Kennedy, Ray Lankester, Henry Morley, and G. D. Thane. Two, namely, Karl Pearson, who was appointed in the same year (1884) to the Chair of Applied Mathematics, and A. F. Murison, who held the Chair of Roman Law, were to remain his colleagues during the forty years of his teaching life at University College.

The life of a professor in a University College in those days was very

different from that to which we are now accustomed. Endowed Chairs were the exception and stipends largely consisted of professors' shares of fees, so that there was a strong inducement to make one's teaching popular rather than profound, a temptation fortunately resisted in most cases, certainly in the case of Hill. Assistants were few, and often paid by the professor. When Hill first took up his duties, the department of Mathematics boasted only a single assistant. The bulk of the routine work of undergraduate teaching, such as the correction of students' exercises, fell upon the professor, and this work Hill, trained as he had been in a hard school, performed with unflagging energy and zeal, and an unselfish devotion which won him the affection and admiration of generation after generation of students. To the last, although many other duties supervened and the general improvement of conditions enabled him to be relieved of such routine work, he insisted on retaining a proportion in his own hands, seeing in it the indispensable means for gaining that personal touch with his students which he felt to be a most important duty and a valued privilege.

It will be readily understood that the prosecution of mathematical research, under such conditions, was no easy task. Nevertheless Hill found time to continue his work on Hydrodynamics, begun at Birmingham, and to publish a few minor papers on Pure Mathematics, largely arising out of his teaching.

It was not until 1888 that he was able to produce really important fresh research work; he then attacked, for the first time, the subject of singular solutions of Differential Equations, a subject for which he always retained a particular liking, for he returned to it in his later years and published a paper on it as late as 1917. In a series of investigations between 1888 and 1893 he dealt fully with the various loci connected with the differential equation of the first order and its complete primitive, extending and consolidating the work of Cayley on the p- and c-discriminants of these equations, and discussing the mode of appearance of the envelope, node-, cusp-, and tac-loci, and important cases of exception. This led him to discuss the more general question of the appearance of loci of singular points in the case of the locus of ultimate intersections of a family of surfaces* ; and here Hill broke entirely fresh ground and obtained a variety of new and valuable results.

Hill's later papers (1916–1921) on Differential Equations may be mentioned here, as they belong logically to the same group. In these papers he considers the classification of integrals of ordinary differential equations of the first order and of partial differential equations of Lagrange's form, and in particular the problem, whether the complete primitive and the singular solution obtained by the ordinary rule do in fact exhaust the possible solutions in the case of the ordinary differential equation of the first order.

In 1891, and again in 1894, he returned for a time to Hydrodynamical

* 'Phil. Trans.,' A, vol. 183, pp. 141–273 (1892).

problems; in the latter year he published the solution for his well-known spherical vortex. This was, however, with the solitary exception of a paper on the Connecting Rod, published in the Proceedings of the Institution of Civil Engineers in 1895, his last excursion into the realm of Applied Mathematics. In view of the success of his earlier Hydrodynamical work, this must remain a matter for regret; but Hill always drew much of his inspiration from his teaching and his Chair was, in effect, limited to Pure Mathematics. Moreover, his mind had a strong bent towards rigour; he was at his best in tasks which demanded critical examination of first principles, and he sometimes expressed, in private conversation, doubts as to the validity of many methods which the applied mathematician is apt to use without question.

It was in 1897 that Hill published, in the Cambridge Philosophical Transactions, his first paper on a subject which he was to make peculiarly his own, namely, the critical re-examination of the Vth and VIth Books of Euclid. He was first led to this by difficulties which he encountered in the attempt to teach beginners the theory of proportion, and, during the next thirty years of his life, much of his mathematical activity was spent in trying to unravel the problems presented by the work of the great Greek geometer, in the form in which it has reached us. In this study of Euclid's Vth Book, Hill followed the tradition of his own great predecessor, Augustus de Morgan, of whom he always spoke with the greatest admiration.

His ideas on the subject are set forth in five papers, published between 1897 and 1922 in the Cambridge Philosophical Transactions, in an edition of the Vth and VIth Books of Euclid, and in a book on the Theory of Proportion, published in 1914. In these he practically reconstructed Euclid's treatment, simplifying many of the proofs, and eliminating unnecessary axioms and definitions. In particular he based his treatment on the view that the definition of *equal* ratios should be adequate for the proofs of all properties of such ratios, and he succeeded in showing that Euclid's appeal to the definition of *unequal* ratios for this purpose was unnecessary. This idea, and the use of the axiom of Archimedes, form the groundwork of his treatment, which is largely permeated by Dedekind's conception of the nature of the irrational number.

It was unfortunate that this work of Hill's coincided with the movement for omitting the study of Euclid from the school curriculum. The philosophical significance of the Vth Book really lies in the fact that it provides a theory of numerical measurement of physical quantities, a fact which was never brought out in the old teaching. Those of us who learnt their mathematics at school on the old lines well remember the feeling of bewilderment which came over us when confronted by that awe-inspiring definition of equal ratios. Had Hill's investigation appeared a few years earlier and been properly appreciated, it might have done much to preserve for the school curriculum one of the finest logical disciplines to be found in elementary mathematics.

The work on the Vth Book did not by any means exhaust Hill's later mathematical activities. In a series of papers on the continuation of Power Series, the last of which dates from 1917, he succeeded in demonstrating that the step by step continuation of a number of such series (including those for $(1+x)^n$, arctan x, arcsin x and the hypergeometric series), round a singularity, actually leads to the well-known interchange of branches, usually obtained directly from the function which the series represents in its first domain of convergence. A number of isolated papers deal with a variety of subjects ranging from the axioms of geometry to the theory of functions. In all Hill's mathematical output comprises nearly fifty original publications.

The mere recital of his contributions to mathematics, however, would give a quite inadequate idea of his position and influence in the academic world. As a teacher he had, to an extraordinary degree, that infinite capacity for taking pains in which Carlyle saw the mark of genius ; and he possessed that rare quality, which students so keenly appreciate, of never slurring over difficulties : time spent on making a demonstration perfect was always to him time well spent. And he showed great sympathy with the occasionally devious mental processes of beginners and would even, sometimes, adapt his demonstrations to them. The writer remembers, in his student days, sending up to him a solution which, alas ! meandered through as many pages as it should have taken lines, arriving at the desired result by a singularly laborious and inelegant process. Hill read patiently and carefully every line, and in the end his only (and characteristic) comment was that it was a " very courageous " solution !

Above all, he loved his students, a feeling which was universally and deeply reciprocated. When he retired in 1924 from the Chair of Mathematics, his friends asked him in what way he would wish them to commemorate his long connection with University College ; and he then, remembering the financial struggle of his early years, expressed the desire that any subscriptions received should be devoted to establishing a Loan Fund by means of which the difficulties of students in straitened circumstances might be temporarily relieved, while their spirit of independence was to be preserved by an undertaking of eventual repayment, so soon as they felt able to do so. It was done according to his wishes, and, indeed, no more fitting memorial could have been found of a life spent in the unselfish service of studious youth.

Hill was one of those who fought for the establishment in London of a real teaching University ; and from the re-constitution of the University in 1900 until 1926, when ill-health compelled his retirement, he was a member of its Senate, in which his balanced judgment, ever-courteous modesty, and, above all, his transparent honesty of purpose and that moral atmosphere which radiated from him and impressed all, even the bitterest opponents of his policy, who came into contact with him, soon gained for him a position of ascendancy. For ten years he was Chairman of the Academic Council, the acknowledged

leader and spokesman of the internal side; and for two years (1909–1911) he was Vice-Chancellor of the University, an honour which has only twice been given to a professor. To his initiative were due many important developments, the full effects of which are only now beginning to be felt, such as the establishment of proper machinery for appointments to Chairs and Readerships and various improvements in the status and qualifications of teachers of the University. His work for the University continually absorbed more and more of his time, and his friends often regretted that he should have accepted such heavy burdens at the expense of his mathematical work. He himself felt this growing difficulty and would often express his sorrow that time was lacking to keep pace in his reading with modern research. That under the circumstances he was able, during the years 1900–1926, to produce as much mathematical work as he did is remarkable; what he would have accomplished, had his scientific activities not been hampered by administrative work, must remain a matter for speculation, but that the loss was great cannot be doubted.

Hill was elected a Fellow of the Society in 1894 and served on the Council in the years 1911–1913. In addition to his London degrees he held the Sc.D. of Cambridge and Hon. LL.D. of St. Andrews. He served on the Council of the London Mathematical Society in 1886 and again for the ten years 1891–1901, being Vice-President in 1894 and 1895. In 1927-8 he was President of the Mathematical Association.

Hill married, in 1892, Minnie Grace, daughter of Marriott Ogle Tarbotton, of Nottingham. There were three children of the marriage, two sons and one daughter, all of whom survive him. Those who were privileged to have access to his home circle appreciated its warm and happy atmosphere, where he found the peace of mind necessary for his arduous tasks. He had the great joy of seeing both his sons pass unscathed through the perils of the War, after winning distinction in the front line with the Royal Flying Corps. His last years were, however, darkened by the loss in 1920 of a beloved and devoted wife and, later, by an illness which culminated, during the last fifteen months of his life, in total blindness. He bore this new trouble with characteristic courage and patience, and, in spite of this handicap, actually completed and delivered, in January, 1928, his second Presidential Address to the Mathematical Association. Almost to the day of his death he continued at work, contending with surprising success against well-nigh insuperable obstacles, and planning a book on the foundations of Geometry.

The end came swiftly and comparatively painlessly on January 11th, 1929.

L. N. G. F.