

CAMILLE JORDAN, 1838-1922.

CAMILLE JORDAN, who died on January 20, 1922, at the age of 84, was one of the great French mathematicians of the nineteenth century. His life was not notably more eventful than that of most mathematicians. He was professor at the École Polytechnique from 1876 to 1912, and also at the Collège de France; a member of the Institute from 1881, and editor of the 'Journal de Mathématiques' ('Liouville's Journal') from 1885. He was elected a Foreign Member of the Society in 1919.

Jordan was a mathematician of great profundity and originality, and left behind him contributions of the first importance to every field in which he worked. He is, no doubt, best known now for his 'Cours d'Analyse,' the celebrated treatise in which, according to the admirable custom of the best French mathematicians, he embodied the substance of his professorial lectures. This work, 'Cours' though it is, is no mere text-book, but contains the substance of the main researches of Jordan's later life, and it is a rare tribute to the fundamental character of these researches that, in so general a treatise, they should seem so perfectly in place.

Jordan's 'Cours' was the first systematic treatise on analysis in which the fundamental problems of the theory of functions were envisaged from a really modern point of view, and it has accordingly played a great part in the education of most of the leading analysts of the day. It was in its second edition, of 1893-1896, that it assumed substantially its final form. The first edition had been comparatively undistinguished. "Dans la précédente édition, où nous tenions à conserver toute la simplicité possible, nous avons glissé un peu rapidement sur les premiers principes . . .", Jordan remarks himself: but now "nous les exposons avec toute la précision et la généralité que nous avons pu, dût-il en résulter quelque complication . . ." The result was a new book and the rise of a new school; for it is fair to attribute to the inspiration of Jordan the beginnings of the movement which, carried on by Hadamard, Borel and Lebesgue, has revolutionised the foundations of modern analysis.

The book, for all its masterfulness, is by no means a very easy one to read. Jordan, I imagine, was no believer in easy roads to the understanding of mathematical truth; he could not shirk a difficulty himself, and he had no intention of allowing his readers to do so. A writer of a text-book, if he be ingenious and competent enough, can often turn a dangerous mathematical obstacle, and lead his army of readers in perfect order to an easy victory, while formidable fortresses lie still unreduced behind him. Jordan invariably scorns such temporary triumphs, knowing too well the disastrous consequences of any later check. Every difficulty, as it arises, must be fairly faced and definitely conquered, and all the ground consolidated in the rear.

Jordan's ideal is, unquestionably, the right one, and it is because he followed it so consistently that his book has been, to the best of mathematical students, so signal an inspiration. To have read it and mastered it is a mathematical education in itself; and it is hardly possible to overstate the influence which it has had on those who, coming to it as I did from the elaborate futilities of 'Tripos' mathematics, have found themselves at last in presence of the real thing. But the ideal is a difficult one, and it is just possible to be too uncompromising in its pursuit. There are times when a little pedagogic ingenuity is innocuous and even useful, and Jordan is apt to push his scorn of it a little too far. He tends to neglect simplicity and symmetry of presentation, even when it might be attained quite easily and without any real surrender of the end in view. It was said of him, by his pupils at the École Normale, that "lorsque M. Jordan rencontre dans un raisonnement quatre quantités jouant enactement le même rôle, il les désigne par u, A', λ, e_3 ."* It is of course a humorous exaggeration, but it has a foundation of justice: "nous avons du moins senti nettement combien M. Jordan se soucie peu de certaines précautions pédagogiques vulgaires . . ."

Among the many individual investigations embodied in the book, there are two which stand out now for originality and for a fruitfulness still quite unexhausted. There is a good deal in the early chapters, the theory of "content," and so on, that has now been superseded; but we shall certainly never forget the function of bounded variation or the continuous Jordan curve. A function $f(x)$, defined in an interval (a, b) , is of bounded variation if, when we divide the interval into parts in any manner by points $x_0 = a, x_1, x_2, \dots, x_n = b$, the sum of the absolute values of the increments $f(x_{i+1}) - f(x_i)$ of the function is less than a number independent of the number or nature of the divisions. It is characteristic of such a function that it can be expressed as the difference of two steadily increasing functions, so that the concept is a generalisation of one of the simplest in mathematics. The generalisation may seem an obvious one, but it is one of the great generalisations which carry simplicity and symmetry in their train. The "monotonic" (increasing or decreasing) function has, for analytical purposes, a fatal defect, namely, that its characteristic property is not invariant for the elementary operations; the sum, for example, of two monotonic functions is not in general monotonic. Functions of bounded variation, while retaining the most essential characteristics of monotonic functions, form a group for these operations, and their introduction gives to the solutions of a large number of important problems a symmetry and unity which would otherwise be quite unattainable. The classical example is that of Dirichlet's conditions for the convergence of a Fourier's series. There is something anomalous in Dirichlet's demonstration, admirable as it is. It is not true that, if Dirichlet's conditions are satisfied by f and g individually,

* I quote from a review of the third edition of the 'Cours' (1915), by M. Lebesgue.

they are also satisfied by $f + g$; while the *result*, if true for two functions, is obviously true also for their sum; the conditions break down and yet the result remains. It was this anomaly that Jordan set himself to remove, and from which his discovery originated.

There is another familiar example of the use of the concept of bounded variation, which shows in a striking manner the connection between different parts of Jordan's work. What is the condition that a continuous curve should have a length? A continuous curve is a set of points (x, y) defined by equations

$$x = x(t), \quad y = y(t),$$

where $x(t)$ and $y(t)$ are continuous functions of a parameter t . The precise definition is Jordan's, and a continuous curve, as so defined, still carries the name of a *Jordan curve*. The *length* of a curve is the limit of the perimeter of an inscribed polygon, and the problem is that of finding the conditions that this limit should exist. There is a complete solution: the curve is rectifiable (possesses a length) if and only if $x(t)$ and $y(t)$ are functions of bounded variation. So perfect is the solution, and so inevitably does the concept of bounded variation intervene, that it is difficult to imagine how the concept came to be invented for an entirely different purpose.

Any mention of the *Jordan curve* suggests inevitably the most famous of all the theorems with which Jordan's name is connected. What is meant by the *inside* or the *outside* of a closed curve? How can these notions be defined precisely, and the rough intuitions of geometrical "common sense" translated into accurate analytical terms? Jordan's solution of the problem is not absolutely complete, but it was he who put the question, stated the solution, and provided the essentials for the first accurate proof. The problem was no doubt suggested to him by his early "topological" researches. A simple closed continuous curve C is a set of points (x, y) in two dimensional space, defined by $x = x(t)$, $y = y(t)$, where t is a parameter which varies, say, from 0 to 2π , and $x(t)$, $y(t)$ are continuous functions such that

$$x(t_1) = x(t_2), \quad y(t_1) = y(t_2)$$

if and only if t_2 is congruent to t_1 to modulus 2π . Such a curve divides the plane into two regions, D_1 and D_2 , of which C is the common boundary. If P and Q are points of the same region, then there is a continuous curve PQ which has no point in common with C ; but if they lie one in D_1 and one in D_2 , then any such curve must meet C once at least. This is Jordan's theorem, the most famous theorem in 'Analysis Situs,' and the source of a whole branch of modern mathematics. Jordan's proof is incomplete, since he assumes the truth of the theorem for a simple polygon, and the success of his attempt to free himself from the pre-suppositions of geometrical intuition is therefore incomplete. The gap has been filled by his successors by the construction,

both of alternative demonstrations in which no such preliminary assumption is required, and of elementary inductive arguments specially applicable to the polygonal case; but the theorem remains justly and permanently associated with his name.

I have written of Jordan as an analyst, and it is no doubt as an analyst that he won the most permanent fame. But even his analysis bears continual witness, in his selection of problems and in the spirit in which he pursues them, to the interests which occupied him in his earlier life. The notion of a group, whether abstract or a group of algebraical or geometrical transformations, is never long absent from his mind, as anyone who has studied his admirable account of the elliptic functions will remember; and his early researches are almost entirely dominated by the theory of groups.

In his '*Traite des Substitutions et des Équations Algébriques*,' published by Gauthier-Villars in 1870, Jordan gave the first comprehensive account of the theory of Galois ("dont tout ceci n'est qu'un commentaire") and its applications to the theory of algebraic equations, with chapters, on a less extensive scale, on geometry and elliptic functions. The treatise is occupied for the most part with groups of linear substitutions. In the last book he solves a problem put by Abel, that of finding all equations of a given degree which are soluble by radicals, and of deciding whether a given equation belongs to this class or not. But there are also very important original contributions to the general theory, in particular concerning primitive groups and factors of composition. A series of memoirs published later complete and extend his results. Only an expert in the theory could venture to pass judgment on Jordan's contributions to it, and I must be content to repeat the verdict of one of the most distinguished of his countrymen, that in his treatise he showed himself "un grand algébriste," with "une rare profondeur d'esprit et une extraordinaire puissance d'abstraction," who "se jouait au milieu des discussions les plus subtiles, se plaisant à aborder les questions dans toute leur généralité, comme s'il craignait que quelque particularité l'empêchat de voir les vraies raisons des choses." It is at any rate a verdict which no one who knows Jordan as an analyst will be inclined to dispute.

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