

## Richard Brauer and the Theory of Algebra



German-American mathematician Richard Brauer, with his wife Ilse Brauer, in 1970, photo: Jacobs, Konrad, CC BY-SA 2.0 DE <<https://creativecommons.org/licenses/by-sa/2.0/de/deed.en>>, via Wikimedia Commons

On February 10, 1901, German and American mathematician **Richard Dagobert Brauer** was born. Brauer worked mainly in abstract algebra, but made important contributions to number theory. He was the founder of modular representation theory.

### Richard Brauer – Early Years

Richard Brauer was born in Charlottenburg, a district of Berlin, Germany, which was not incorporated into the city until 1920, to Max Brauer, a well-off businessman in the wholesale leather trade, and his wife Lilly Caroline. Alfred Brauer was Richard's older brother and both were interested in science and mathematics, but Alfred was injured in combat in World War I,

while Richard was young enough to avoid being drafted into the army. As a boy, Richard dreamt of becoming an inventor, and in February 1919 enrolled in Technische Hochschule Berlin-Charlottenburg. He soon transferred to University of Berlin. Except for the summer of 1920 when he studied at University of Freiburg, he studied in Berlin, being awarded his Ph.D. (with distinction) in 1926. Issai Schur conducted a seminar and posed a problem in 1921 that Alfred and Richard worked on together, and published a result. The problem also was solved by Heinz Hopf at the same time. Richard wrote his thesis under Schur, providing an algebraic approach to irreducible, continuous, finite-dimensional representations of real orthogonal (rotation) groups.

## A Family of Mathematicians

Before the award of his doctorate, however, Brauer had married Ilse Karger in September 1925. They had been a fellow students in one of Schur's courses on number theory.[\[1\]](#) Their sons George Ulrich (b 1927) and Fred Gunther (b 1932) also became mathematicians. Brauer began his teaching career in Königsberg (now Kaliningrad) working as Konrad Knopp's assistant. Brauer expounded central division algebras over a perfect field while in Königsberg; the isomorphism classes of such algebras form the elements of the Brauer group he introduced.

## Emigration

Political events forced Brauer's family to move. When the Nazi Party took over in 1933, the Emergency Committee in Aid of Displaced Foreign Scholars took action to help Brauer and other Jewish scientists. Brauer was offered an assistant professorship at University of Kentucky. He accepted the offer, and by the end of 1933 he was in Lexington, Kentucky, teaching in English. Ilse followed the next year with George and Fred; brother Alfred made it to the USA in 1939, but their sister Alice was killed in the Holocaust.

## Academic Career

Hermann Weyl invited Richard to assist him at Princeton's Institute for Advanced Study in 1934,[\[10\]](#) where together with Nathan Jacobson, he edited Weyl's lectures *Structure and Representation of Continuous Groups*. Collaboration between Brauer and Weyl on several projects followed, in particular a famous joint paper on spinors published in 1935 in the *American Journal of Mathematics*. This work was to provide a background for the work of Paul Dirac in his exposition of the theory of the spinning electron within the framework of quantum mechanics.[\[1,8\]](#) Through the influence of Emmy Noether[\[9\]](#), Richard was invited to University of Toronto to take up a faculty position and remained there until 1948, when he left to join the faculty at the University of Michigan. During this time Brauer developed some of his most impressive theories, carrying the work of Georg Frobenius into a whole new setting, in particular the work on group characters Frobenius published in 1896. Brauer carried Frobenius's theory of ordinary characters, where the characteristic of the field does not divide the order of the group, to the case of modular characters, where the characteristic does divide the group order.[\[1,4\]](#)

## Modular Representation Theory

Brauer also studied applications to number theory. With his graduate student Cecil J. Nesbitt he developed modular representation theory, published in 1937. Modular representation theory is a branch of mathematics, and that part of representation theory that studies linear

representations of finite groups over a field  $K$  of positive characteristic. In 1948 Richard and Ilse moved to Ann Arbor, Michigan where he and Robert M. Thrall contributed to the program in modern algebra at University of Michigan. With his graduate student K. A. Fowler, Brauer proved the Brauer–Fowler theorem. In mathematical finite group theory, the Brauer–Fowler theorem, states that if a group  $G$  has even order  $g > 2$  then it has a proper subgroup of order greater than  $g^{1/3}$ . Donald John Lewis was another of his students at UM.

## The Brauer–Suzuki Theorem

In 1952 Brauer joined the faculty of Harvard University. He was chairman of the department from 1959 to 1963. The Brauer–Fowler theorem later provided significant impetus towards the classification of finite simple groups, for it implied that there could only be finitely many finite simple groups for which the centralizer of an involution (element of order 2) had a specified structure. Brauer applied modular representation theory to obtain subtle information about group characters, particularly via his three main theorems. These methods were particularly useful in the classification of finite simple groups with low rank Sylow 2-subgroups. The Brauer–Suzuki theorem showed that no finite simple group could have a generalized quaternion Sylow 2-subgroup, and the Alperin–Brauer–Gorenstein theorem classified finite groups with wreathed or quasidihedral Sylow 2-subgroups.

## Later Years

Before retiring in 1971 he taught aspiring mathematicians such as Donald Passman and I. Martin Isaacs. Brauer was to spend the rest of his life working on the problem of classifying the finite simple groups. He died before the classification was complete but his work provided the framework of the classification which was completed only a few years later.[1] Richard Brauer died on April 17, 1977, aged 76.

## References and Further Reading:

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- [4] [Ferdinand Georg Frobenius and Group Theory](#), SciHi blog, August 3, 2016.
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