Jacques Hadamard and the Description of Mathematical Thought

Jacques Hadamard (1865-1963)

On December 8, 1865, French mathematician **Jacques Salomon Hadamard** was born. Hadamard made major contributions in number theory, complex function theory, differential geometry and partial differential equations. Moreover, he is also known for his description of the mathematical though process in his book *Psychology of Invention in the Mathematical Field*.

"It is important for him who wants to discover not to confine himself to one chapter of science, but to keep in touch with various others."
— Jacques Hadamard, [13]

Jacques Hadamard – Early Years

Jacques Hadamard was born the son of a teacher, Amédée Hadamard, of Jewish descent, and Claire Marie Jeanne Picard, in Versailles, France. He attended the Lycée Charlemagne and Lycée Louis-le-Grand, where his father taught. This was an unfortunate time for a child to be growing up in Paris, since the Franco-Prussian War which began on 19 July 1870 went badly for France and on 19 September 1870 the Prussians began a siege of Paris. This was a desperate time for the inhabitants, who were suffering from hunger. Paris surrendered on 28 January 1871 and the Treaty of Frankfurt, signed on 10 May 1871, was a humiliation for France. Between the surrender and the signing of the treaty there was essentially a civil war in Paris and the Hadamards' house was burnt down.[1]

Studying Mathematics

In 1884 Hadamard entered the École Normale Supérieure, having been placed first in the entrance examinations both there and at the École Polytechnique. His teachers included amongst others mathematician and historian Paul Tannery, mathematician Charles Hermite [1], and mathematician Charles Émile Picard. While undertaking research for his doctorate he worked as a school teacher. Although his research went extremely well, his teaching was less appreciated, probably because he demanded more of his pupils than their abilities allowed. Hadamard obtained his doctorate in 1892 and in the same year was awarded the Grand Prix des Sciences Mathématiques for his essay on the Riemann zeta function.

Hadamard Matrices and the Prime Number Theorem

In June of the same year, Hadamard married Louise-Anna Trénel, who had known each other from childhood. The couple moved to Bordeaux the following year when Hadamard was appointed as a lecturer at the University, where he proved his celebrated inequality on determinants, which led to the discovery of Hadamard matrices when equality holds. He published 29 papers during these four years, but they are remarkable more for their depth and the range of the topics which they covered rather than their number. In 1896 he made two important contributions: he proved the prime number theorem, using complex function theory (also proved independently by Charles Jean de la Vallée-Poussin); and he was awarded the Bordin Prize of the French Academy of Sciences for his work on geodesics in the differential geometry of surfaces and dynamical systems.

Academic Career

In the same year he was appointed Professor of Astronomy and Rational Mechanics in Bordeaux. His foundational work on geometry and symbolic dynamics continued in 1898 with

the study of geodesics on surfaces of negative curvature. For his cumulative work, he was awarded the Prix Poncelet in 1898.

The Dreyfus Affair

After the Dreyfus affair [3], which involved him personally because his wife was related to Dreyfus, Hadamard became politically active and a staunch supporter of Jewish causes though he professed to be an atheist in his religion. Dreyfus was also of Jewish descent and embarked on a military career. Working at the War Ministry, he was accused of selling military secrets to the Germans and he was sentenced to life imprisonment. Although his trial had been highly irregular, the anti-Semitic views of many people made the verdict popular. Forged documents and cover-ups soon showed that the legal process had been suspect. At first Hadamard, like many people, assumed that Dreyfus was guilty. However after moving to Paris in 1897 he began to discover how evidence against Dreyfus had been forged.[1] Hadamard became a leading crusader to reopen the case against Dreyfus, who happened to be a relative of his wife. Eventually, Dreyfus was retried, found guilty again, and pardoned. Hadamard would not accept this and was among those who continued to press the government to clear Dreyfus's name—a result finally achieved in 1906.[4]

The Method of Descent

In 1897 he moved back to Paris, holding positions in the Sorbonne and the Collège de France, where he was appointed Professor of Mechanics in 1909. In addition to this post, he was appointed to chairs of analysis at the École Polytechnique in 1912 and at the École Centrale in 1920. In Paris Hadamard concentrated his interests on the problems of mathematical physics, in particular partial differential equations, the calculus of variations and the foundations of functional analysis. He introduced the idea of well-posed problem and the method of descent in the theory of partial differential equations, culminating in his seminal book on the subject, based on lectures given at Yale University in 1922.

Honors and Later Life

"Practical application is found by not looking for it, and one can say that the whole progress of civilization rests on that principle."
– Jacques Hadamard, [13]

Among many other honors, Hadamard was elected to the French Academy of Sciences in 1916, in succession to Poincaré. At the beginning of the Second World War Hadamard escaped to southern France in 1940. The Vichy government permitted him to leave for the United States in 1941 and he obtained a visiting position at Columbia University in New York. In 1945 he published his reflections and investigations of the mathematical mind, entitled *The Psychology of Invention in the Mathematical Field*. Having lost his two older sons in World War I and another during World War II, he became active in international peace movements.[4] After the war, Hadamard returned to France, where he died in 1963, aged 97. Jacques Hadamard's body of mathematical work is impressive both in its depth and in its breadth. In particular, his work transformed the theory of functions, contributed to the creation of functional analysis, and breathed new life into the theory of partial differential equations. Moreover, the influence of his legacy on the development of analysis in the 20th century and on the Bourbaki group is impossible to overstate.[6]

References and Further Reading:

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- [4] Jacques-Solomon Hadamard, at Britannica Online
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- [13] Jacques Hadamard, *An Essay on the Psychology of Invention in the Mathematical Field* (1954)
- [14] Timeline for Jacques Hadamard, via Wikisource