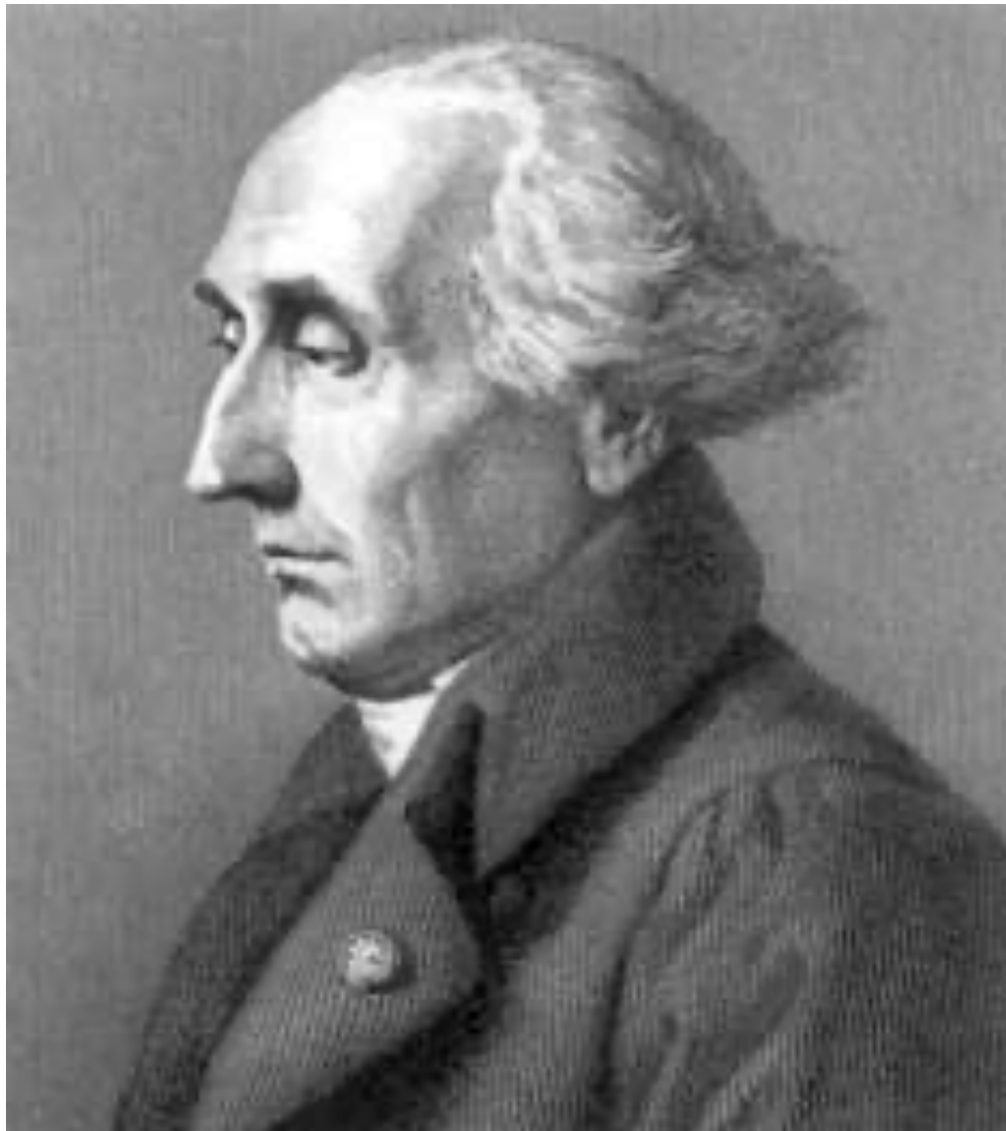# Joseph-Louis Lagrange and the Celestial Mechanics

Joseph-Louis Lagrange (1736-1813)

On April 10, 1813, Italian mathematician and astronomer **Joseph-Louis Lagrange** passed away. Lagrange made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.

"I cannot say whether I will still be doing geometry ten years from now. It also seems to me that the mine has maybe already become too deep and unless one finds new veins it might have to be abandoned. Physics and chemistry now offer a much more glowing richness and much easier exploitation. Also, the general taste has turned entirely in this direction, and it is not impossible that the place of Geometry in the Academies will someday become what the role of the Chairs of Arabic at the universities is now."
– Joseph Louis Lagrange, Letter to d'Alembert (1781)

A Self Taught Mathematician

Lagrange was born on January 25, 1736 as Giuseppe Ludovico Lagrangia in Turin, previously capital of the duchy of Savoy, but became the capital of the kingdom of Sardinia in 1720. His father was Giuseppe Francesco Lodovico Lagrangia, Treasurer of the Office of Public Works and Fortifications in Turin, but the family suffered considerable financial losses through speculation. Lagrange attended the Turin College, where he showed his first mathematical interest at the age of seventeen. His father wanted him to become a lawyer, but at school Lagrange eventually became more interested in mathematics. However, it was not until he was seventeen that he showed any taste for mathematics – his interest in the subject being first excited by a paper by Edmond Halley which he came across by accident. Within one year he taught himself all the knowledge of a fully qualified mathematician of his time.[4,5]

Ballistics, Differential Equations and Calculus of Variations

On 23 July 1754, at age 18, Lagrange published his first mathematical work which took the form of a letter written in Italian to Giulio Fagnano. Charles Emmanuel III appointed Lagrange to serve as the "*Sostituto del Maestro di Matematica*" (mathematics assistant professor) at the Royal Military Academy of the Theory and Practice of Artillery in 1755, where he taught courses in calculus and mechanics to support the Piedmontese army's early adoption of the ballistics theories of Benjamin Robins and Leonhard Euler.[6] In that capacity, Lagrange was the first to teach calculus in an engineering school. According to Alessandro Papacino D'Antoni, the academy's military commander and famous artillery theorist, Lagrange unfortunately proved to be a problematic professor with his oblivious teaching style, abstract reasoning, and impatience with artillery and fortification-engineering applications. Here he published his first scientific papers on differential equations and calculus of variations. In 1757 he was one of the founders of the Turin Academy, and most of his early writings are to be found in the five volumes of its transactions, usually known as the *Miscellanea Taurinensia*.

The Calculus of Variations

Lagrange is one of the founders of the calculus of variations. Starting in 1754, he worked on the problem of tautochrone, discovering a method of maximising and minimising functionals in a way similar to finding extrema of functions. Lagrange wrote several letters to Leonhard Euler between 1754 and 1756 describing his results. He outlined his "δ-algorithm", leading to the Euler–Lagrange equations of variational calculus and considerably simplifying Euler's earlier analysis. Lagrange also applied his ideas to problems of classical mechanics, generalising the results of Euler and Maupertuis. Euler was very impressed with Lagrange's

results. It has been stated that "*with characteristic courtesy he withheld a paper he had previously written, which covered some of the same ground, in order that the young Italian might have time to complete his work, and claim the undisputed invention of the new calculus*"; however, this chivalric view has been disputed. Lagrange published his method in two memoirs of the Turin Society in 1762 and 1773.

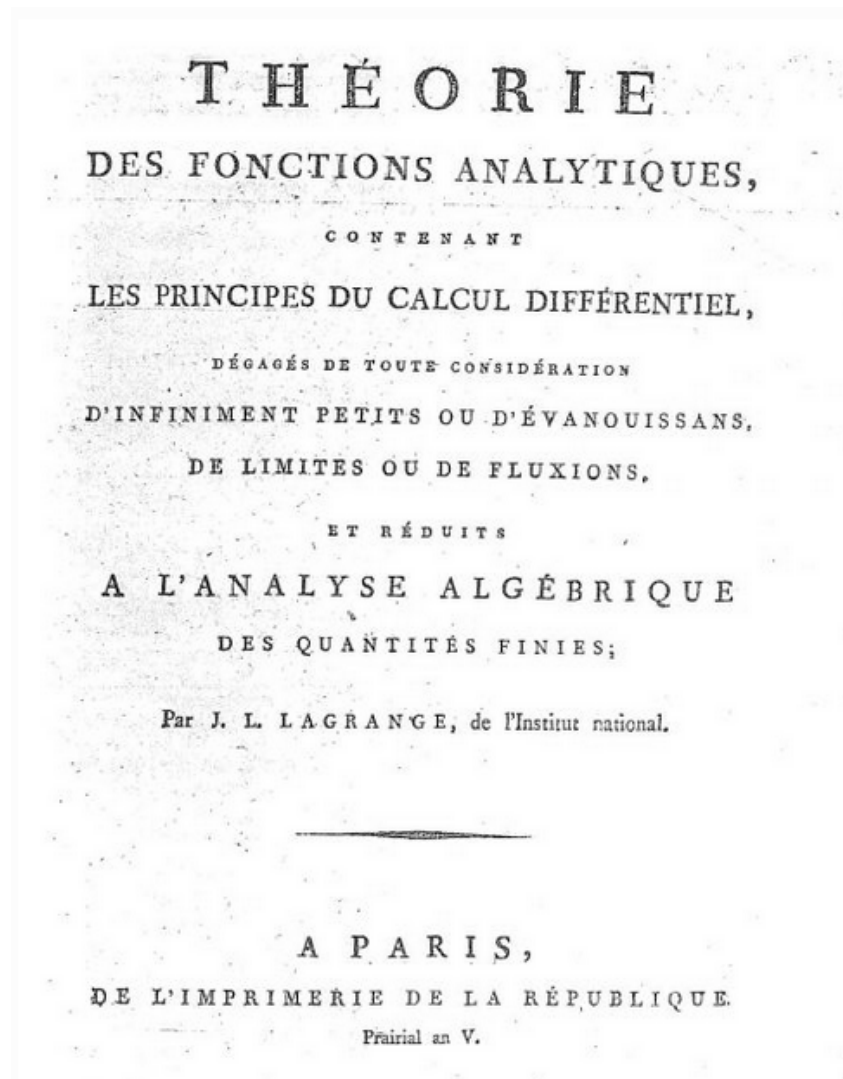## The Libration of the Moon

In 1756, Euler and Maupertuis, seeing his mathematical talent, tried to persuade him to come to Berlin, but Lagrange had no such intention and shyly refused the offer. The Académie des Sciences in Paris announced its prize competition for 1764 in 1762. The topic was on the libration of the Moon, that is the motion of the Moon which causes the face that it presents to the Earth to oscillate causing small changes in the position of the lunar features. Lagrange entered the competition, sending his entry to Paris in 1763 which arrived there not long before Lagrange himself. In 1765, Lagrange entered, later that year, for the Académie des Sciences prize of 1766 on the orbits of the moons of Jupiter.[1] The prize was again awarded to Lagrange, and he won the same distinction in 1772, 1774, and 1778.[2]

Analytical Mechanics

Finally, in 1765, d'Alembert interceded on Lagrange's behalf with Frederick of Prussia and by letter, asked him to leave Turin for a considerably more prestigious position in Berlin.[7] Lagrange was finally persuaded and he spent the next twenty years in Prussia, where he produced not only the long series of papers published in the Berlin and Turin transactions, but also his monumental work, the *Mécanique analytique*. Here he dealt with problems of astronomy, but also with partial differential equations and questions of geometry and algebra. Lagrange was a favourite of the king, who used frequently to discourse to him on the advantages of perfect regularity of life. The lesson went home, and thenceforth Lagrange studied his mind and body as though they were machines, and found by experiment the exact amount of work which he was able to do without breaking down. Every night he set himself a definite task for the next day, and on completing any branch of a subject he wrote a short analysis to see what points in the demonstrations or in the subject-matter were capable of improvement. He always thought out the subject of his papers before he began to compose them, and usually wrote them straight off without a single erasure or correction.

A Prolific Mathematician

Lagrange's productivity in those years was prodigious: he published papers on the three-body problem, which concerns the evolution of three particles mutually attracted according to Sir Isaac Newton's law of gravity; differential equations; prime number theory; the fundamentally important number-theoretic equation that has been identified (incorrectly by Euler) with John Pell's name;[8] probability; mechanics; and the stability of the solar system. In his long paper "*Réflexions sur la résolution algébrique des équations*" (1770; "*Reflections on the Algebraic Resolution of Equations*"), he inaugurated a new period in algebra and inspired Évariste Galois to his group theory.[2]

THÉORIE
DES FONCTIONS ANALYTIQUES,

CONTENANT

LES PRINCIPES DU CALCUL DIFFÉRENTIEL,

DÉGAGÉS DE TOUTE CONSIDÉRATION

D'INFINIMENT PETITS OU D'ÉVANOUISSANS,

DE LIMITES OU DE FLUXIONS,

ET RÉDUITS

A L'ANALYSE ALGÉBRIQUE

DES QUANTITÉS FINIES;

Par J. L. LAGRANGE, de l'Institut national.

A PARIS,

DE L'IMPRIMERIE DE LA RÉPUBLIQUE.

Prairial an V.

Joseph-Louis Lagrange, Théorie des fonctions analytiques (1797)

Lethargy and Revolution

In 1786, following Frederick's death, Lagrange received invitations from states including Spain and Naples, and he accepted the offer of Louis XVI to move to Paris. In France he was received with every mark of distinction and special apartments in the Louvre were prepared for his reception, and he became a member of the French Academy of Sciences. At the beginning of his residence in Paris he was seized with an attack of melancholy, and even the printed copy of his *Mécanique* on which he had worked for a quarter of a century lay for more than two years unopened on his desk. Curiosity as to the results of the French revolution first stirred him out of his lethargy, a curiosity which soon turned to alarm as the revolution developed.

The Reform of the Metric System

The Revolution, which began in 1789, pressed Lagrange into work on the committee to reform the metric system. When the great chemist Antoine-Laurent Lavoisier was guillotined, Lagrange commented, "*It took them only an instant to cut off that head, and a hundred years may not produce another like it.*"[2] Though Lagrange had been preparing to escape from

France while there was yet time, he was never in any danger; different revolutionary governments (and at a later time, Napoleon) loaded him with honours and distinctions.

Professor of Analysis

Lagrange was considerably involved in the process of making new standard units of measurement in the 1790s. He was offered the presidency of the Commission for the reform of weights and measures (la Commission des Poids et Mesures) when he was preparing to escape. And after Lavoisier's death in 1794, it was largely owing to Lagrange's influence that the final choice of the unit system of metre and kilogram was settled and the decimal subdivision was finally accepted by the commission of 1799. Lagrange was also one of the founding members of the Bureau des Longitudes in 1795. The École Polytechnique was founded on 11 March 1794 and opened in December 1794. Lagrange was its first professor of analysis, appointed for the opening in 1794. In 1795 the École Normale was founded with the aim of training school teachers. Lagrange taught courses on elementary mathematics there.

Further Achievements

As for the solution of polynomial equations, Lagrange, in his *Reflexions sur la Theorie Algebriques des Equations* of 1770, tried to solve algebraically polynomial equations of degree five and higher starting with the procedure used by Cardano. He tried to generalize by considering permutations of the roots. However, he was unsuccessful with that and he was thus forced to abondon his quest. Nevertheless, his work did form the foundation on which all nineteenth-century work on the algebraic solutions of equations was based, especially that of Cauchy who was able later to take the theory of permutations to a deeper level.[3]

Later Years

Finally, Lagrange's commitment to the necessity of an algebraic foundation for the calculus led him to the major accomplishment of the *Fonctions Analytiques*, in which he studied functions by means of their power series expansions. He believed that every function could be expanded into a power series, where for Lagrange, a function was defined as follows:One names a function of one or several quantities any mathematical expression in which the quantities enter in any manner whatever, connected or not with other quantities which one regards as having given and constant values, whereas the quantities of the function may take any possible values.[3]

There is a nice anecdote about Lagrange's later life. Lagrange, in one of the later years of his life, imagined that he had overcome the difficulty of the parallel axiom. He went so far as to write a paper, which he took with him to the Institute, and began to read it. But in the first paragraph something struck him which he had not observed. He muttered:

Il faut que j'y songe encore.

And he put the paper in his pocket.[3]

Joseph-Louis Lagrange, Senator, Count of the Empire, Grand Officer of the Legion of Honour, Grand Cross of the Imperial Order of the Reunion, Member of the Institute and the Bureau of Longitude, died in Paris on 10 April 1813.

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