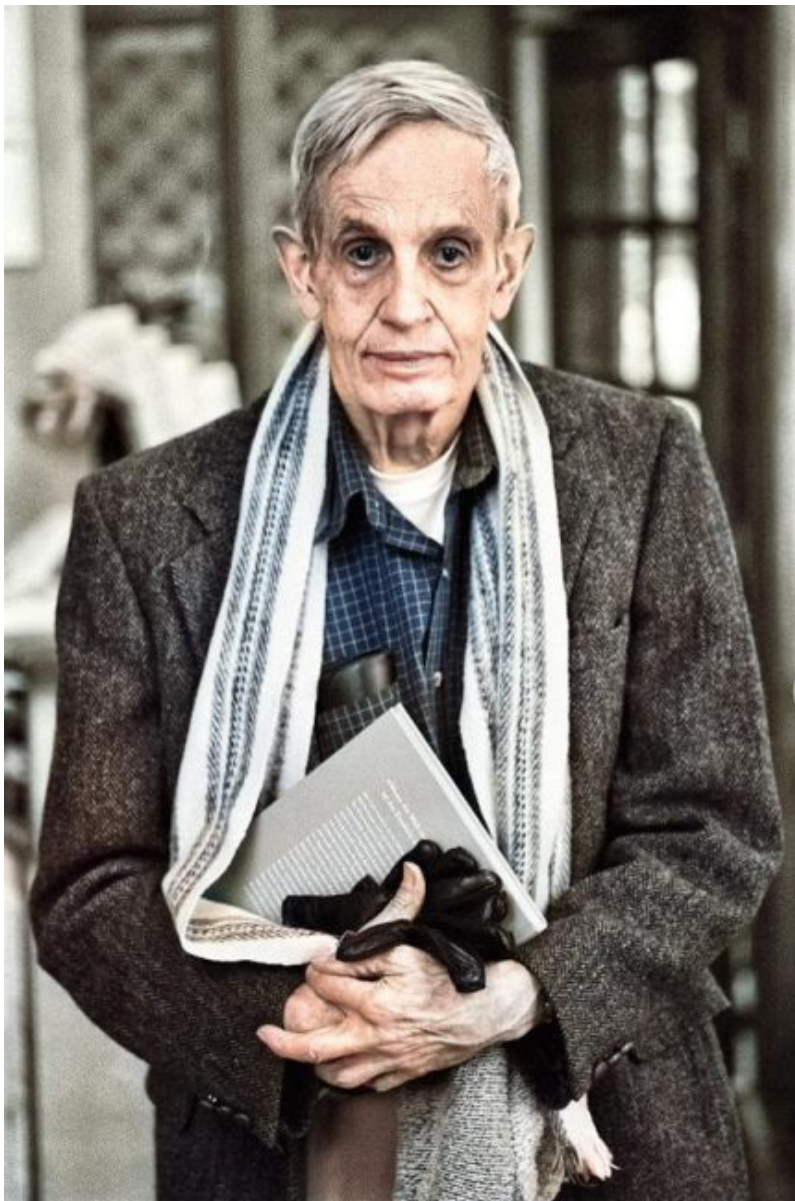


## The Beautiful Mind of John Forbes Nash



John Forbes Nash Jr. (1928-2015)

On June 13, 1928, American mathematician **John Forbes Nash Jr.** was born. Nash made fundamental contributions to game theory, differential geometry, and the study of partial differential equations. His work has provided insight into the factors that govern chance and decision-making inside complex systems found in everyday life. John Nash is the only person to be awarded both the Nobel Memorial Prize in Economic Sciences and the Abel Prize.

“*You don’t have to be a mathematician to have a feel for numbers.*”  
– John Forbes Nash Jr.

## **Youth and Education**

Nash was born on June 13, 1928, in Bluefield, West Virginia. His father, John Forbes Nash, was an electrical engineer for the Appalachian Electric Power Company. His mother, Margaret Virginia (née Martin) Nash, had been a schoolteacher before she was married. From 1945 to 1948 Nash studied at the Carnegie Institute of Technology in Pittsburgh, where he received his Bachelor’s degree in 1945 and his Master’s degree in 1948. Originally he wanted to become an engineer like his father, but developed a great passion for mathematics. Still in Pittsburgh his interest in game theory began, whose solution John von Neumann and Oskar Morgenstern had left open in their book *Theory of Games and Economic Behavior*, 1944.[\[1, 2\]](#)

## **Game Theory and the Nash Equilibrium**

Nash received his doctorate from Princeton University in 1950 from the mathematician Albert W. Tucker. The work entitled *Non-cooperative Games* extended the game theory of Morgenstern and von Neumann to include the so-called Nash equilibrium. Nash proved that this equilibrium – deviating from the solutions – also exists for non-zero-sum games and for more than two players. The Nash equilibrium describes a combination of strategies in non-cooperative games, where each player chooses exactly one strategy from which it makes no sense for any player to deviate from his chosen strategy as the only one. In a Nash equilibrium every player agrees with his strategy choice afterwards, he would hit it again in the same way. The strategies of the players are therefore the best answers for each other. The Nash equilibrium is an elementary solution concept of game theory.

## **The Nobel Prize in Economics**

The importance of this work from 1950 was only later recognized in connection with the further development of game theory and earned him the Nobel Prize for Economics in 1994. Von Neumann himself was not very impressed at the time at a meeting with Nash; he considered the result to be trivial and mentioned it only indirectly in the introduction to the new edition of his book *Morgenstern on game theory* from 1953. Nash himself also regarded the work more as a by-product in comparison to his later works.

## **Nash Manifolds and Nash Functions**

In 1952 his work on real algebraic manifolds appeared, which he himself regarded as his perfect work. The idea was to approximate each multiplicity by an algebraic variety (much easier to handle and describable by polynomials), possibly by moving to spaces of much higher dimension. In this context, Nash manifolds and Nash functions are named after him. After receiving his doctorate, Nash increasingly turned to analysis, in particular differential geometry and partial differential equations. He proved that every Riemann multiplicity isometric in the

Euclidean vector space can be embedded (Nash Embedding theorem). The question of whether this is possible was already asked by Bernhard Riemann, and the common opinion in the 1950s was that this was not the case. The result of Nash came unexpectedly and had far-reaching consequences. [3]

## **Game Theory and Cold War**

From 1950 onwards, Nash worked for four years in the summer months at Rand Corporation on secret research related to applications of game theory to strategic situations in the Cold War. From 1951 to 1953, Nash was a Moore instructor at the Massachusetts Institute of Technology, where he was assistant professor from 1953 and associate professor from 1957 to 1959. In 1955, he submitted a proposal for an encryption procedure to the National Security Agency, but received a rejection. In 1958 he published (in parallel with Ennio De Giorgi, but independently of him) a solution to the regularity problem of partial differential equations, which David Hilbert had included in 1900 in his well-known list of the largest open problems in mathematics (19th problem).[4] The results became known as De Giorgi's and Nash's theorem and have far-reaching consequences for the theory of partial differential equations.

## **Paranoid Schizophrenia**

At the end of the 1950s Nash was widely recognized as a leading mathematician, which was also reflected in an article in Forbes Magazine, and he was nominated for the Fields Medal in 1958, particularly for his work on Hilbert's 19th problem simultaneously with de Giorgi. He was third in the final evaluation behind Klaus Roth and René Thom, who finally received the Fields Medal in 1958. At MIT he was on the verge of a full professorship when the first signs of Nash's disease became apparent in 1959. In May 1959 he was diagnosed with paranoid schizophrenia. According to Nash biographer Sylvia Nasar, Nash was now increasingly showing anti-Semitic tendencies and prone to violence. Nash gave up his position at MIT and after a short stay in hospital went first to Paris and Geneva in 1959/60, where he saw himself as a citizen of the world and exile. In 1961, his wife Alicia Lardé and his mother were forced to send Nash to a mental hospital (Trenton State Hospital). Here he was treated by insulin shock therapy, which put him in an artificial coma. He recovered and was able to attend a conference on game theory in 1961.

## **Slow Recovery**

In 1964 his schizophrenia became so severe that he had to be admitted to a psychiatric clinic (the private Carrier Clinic) for a longer period of time. During the next 20 years he was repeatedly in clinics for relapses with interruptions. Between 1966 and 1996, as a result of his illness, he did not produce any publications. Before that, however, some outstanding works were published. The 1960s saw the emergence of an idea in the theory of the resolution of singularities in algebraic geometry known as *Nash Blowing Up* (so called by Heisuke Hironaka, to whom Nash gave the idea orally) and some influential papers on partial differential equations. In the 1970s to 1990s he lived in Princeton, where he was regularly seen on campus. While at first he attracted the students' attention with strange messages he left behind, from the early 1990s mathematicians in Princeton (like Peter Sarnak) increasingly noticed that he had regained some of his old problem-solving skills. In his last years, he increasingly turned to monetary theory, arguing for index money.

Nash died with his wife in May 2015 in a traffic accident on the New Jersey Turnpike; they were in a taxi on their way home from receiving the Abel Award. Both were not strapped on and were thrown out of the vehicle. The 2001 feature *A Beautiful Mind*, starring Russell Crowe, tells the story of Nash's ingenious designs, illness and recovery; the film won four Oscars in 2002. The script is based on Sylvia Nasar's 1998 biography of the same name.

### References and Further Reading:

- [1] [John von Neumann – Game Theory and the Digital Computer](#), SciHi Blog
- [2] [Oskar Morgenstern and the Game Theory](#), SciHi Blog
- [3] [Bernhard Riemann's novell approaches to Geometry](#), SciHi Blog
- [4] [David Hilbert's 23 Problems](#), SciHi Blog
- [5] O'Connor, John J.; Robertson, Edmund F., "[John Forbes Nash Jr.](#)", MacTutor History of Mathematics archive, University of St Andrews.
- [6] [Home Page of John F. Nash Jr. at Princeton](#)
- [7] Fisher, Len (May 25, 2015). "[John Nash obituary](#)". *The Guardian*.
- [8] [Biography of John Forbes Nash Jr.](#) from the Institute for Operations Research and the Management Sciences
- [9] [John Forbes Nash Jr at Wikidata](#)
- [10] [The Extraordinary Theorems of John Nash – with Cédric Villani](#), [The Royal Institution](#) @ youtube
- [11] Goode, Erica (May 24, 2015). "[John F. Nash Jr., Math Genius Defined by a 'Beautiful Mind,' Dies at 86](#)". *The New York Times*.
- [12] Milnor, John (1998). "[John Nash and "A Beautiful Mind"](#)". *Notices of the AMS*. Providence, Rhode Island: American Mathematical Society. **25** (10): 1329–1332.
- [13] [Timeline of Game Theorists](#), via DBpedia and Wikidata