

Niccoló Tartaglia and how to solve Cubic Equations



Niccolò Fontana Tartaglia (1499/1500 – 13. December 1557)

On December 13, 1557, Italian Renaissance mathematician and engineer **Niccolò Fontana Tartaglia** passed away. Tartaglia is best known today for his contributions in solving cubic equations. He published many books, including the first Italian translations of Archimedes [1] and Euclid,[2] and an acclaimed compilation of mathematics.

*“When the cube and the things together
Are equal to some discrete number,
Find two other numbers differing in this one.
Then you will keep this as a habit
That their product shall always be equal
Exactly to the cube of a third of the things.
The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing. “*

– Niccolò Tartaglia, the poem in which he revealed the secret of solving the cubic to Cardano

Tartaglia’s Origins

In his book *Quesiti et Inventioni Diverse (Various Tasks and Inventions)*, published in 1546, Tartaglia answered questions about his origins and childhood in a dialogue: his father was a letter carrier on horseback and was called Michele. He does not know a family name. Asked why then he called himself *Tartaglia*, he recounted that in February 1512, when the French sacked his native city of Brescia and inflicted a terrible massacre, a soldier inflicted three wounds on his head and two on his face with sword blows, making him look like a monster if his full beard did not hide it. Among the injuries was one across the mouth and teeth, through which he could not speak properly for a time, but only stutter. Therefore, the children gave him the nickname *Tartaglia (stutterer)*, which he kept as a souvenir of his misfortune. At that time he was about 12 years old. That is, he was born around the year 1500. In a document of 1529 a Nicolo from Brescia, master of arithmetic, certainly Tartaglia, is mentioned with an age of 30 years. This gives 1499 as the year of birth.

An Autodidact

At the age of 14, as he further reported, Niccolò spent two weeks in a writing school learning the ABCs up to K. Then he ran out of money and stole a ready-made alphabet, with the help of which he taught himself the remaining letters. ...and so, from that day on, I was never again with any teacher, but only in the company of a daughter of poverty called Diligence. In other words, all his knowledge of mathematics and military science he acquired as an autodidact in self-instruction. Tartaglia left Brescia around 1516, went via Crema, Bergamo and Milan to Verona, where he lived from about 1521 to 1534, and then moved to Venice, where he lived, with the exception of a year and a half stay in Brescia in 1548/49, until his death in 1557.

A Commercial Calculator and Commercial Tutor

Tartaglia earned his living as a mostly commercial calculator and private tutor. Occasionally he gave lectures and, during the 18 months in Brescia, lectures on Euclid’s *Elements*, for which he received only a fraction of the fixed fee. A surviving list of his poor legacy shows the paucity in which one of the great mathematicians of the Italian Renaissance lived.

Tartaglia's Work

In February 1543, Tartaglia published the first translation of Euclid's *Elements* into Italian, under the title *Euclide Megarense Philosopho: only introduction to the mathematical sciences... after the two translations*. The title is incorrect because Euclid of Megara was a philosopher who lived a century before the mathematician Euclid of Alexandria who is actually meant. The two translations used for this by Tartaglia, both Latin, were by Giovanni Campano, Latinized Johannes Campanus (1220-1296), printed in 1482, and by Bartolomeo Zamberti or Zamberto (1473-after 1543), printed in 1505. As a connoisseur of Euclid, Tartaglia was an expert on the fundamentals of geometry.

The Cubic Equations

Tartaglia became famous not so much because of his books, but because he was involved in a heated dispute about the solution of cubic equations. Today one speaks of a single cubic equation $x^3 + ax^2 + bx + c = 0$, where a , b and c can also be negative or 0, but at that time negative numbers were rejected. Therefore, 13 different cubic equations were distinguished: seven complete ones in which all powers are represented, three without a linear member and three without a quadratic member, namely in modern notation $x^3 + px = q$, $x^3 = px + q$ and $x^3 + q = px$. The third of these equations has a negative principal solution and was therefore usually not treated.

First Tries

For a long time one had searched for a solution of the cubic equations. Finally, the lecturer of the University of Bologna Scipione dal Ferro (1465-1526) had found the solution of the first two equations without a quadratic member around 1505 or 1515, but had not published it. Such knowledge was, in fact, extremely valuable as an offensive or defensive weapon at a time when a university teacher's reappointment and salary depended on how he performed in the frequent public scholarly contests in which the two opponents set each other tasks and problems.

The Solution

Arithmetic masters also engaged in such mathematical battles, and so in early January 1535 Tartaglia and his Venetian rival Antonio Maria Fior also set each other 30 tasks to be solved within 40 or 50 days. Fior, as a student of dal Ferro, boasted of having the solution of the cubic equation (modern) $x^3 + px = q$. All of Fior's 30 problems were of this form. Thereupon, Tartaglia exerted himself and found the solution rule on February 12, 1535, and one day later also the one for the equation (modern) $x^3 = px + q$. According to him, he solved all of Fior's problems within two hours, while Fior could not solve a single one.

Cardano and the Publication

In the *Quesiti*, Tartaglia reports that on January 2, 1539, a bookseller from Milan appeared at his house. He had been sent by the physician Gerolamo Cardano (1501-1576),^[3] who was considered a very great mathematician, was publicly reading Euclid in Milan, and was now having a work printed on the practice of arithmetic and geometry and on algebra. And because he had heard that Tartaglia, in a contest with Master Fior, had solved all 30 problems on the equation *Cosa e Cubo* (the unknown and the cube) equal to a number within two hours, "he asks that you send him this rule you have discovered, and if it is agreeable to you, he will

publish it in his present work under your name, and if it is not agreeable to you that he should publish it, he will keep it secret.” Tartaglia’s reply, “Tell his Excellency that you forgive me, but if I want to publish this invention of mine, it will be in my own works and not in those of others.”

But Cardano did not let up. He pressed Tartaglia by letter and invited him to Milan under the pretext that the Spanish governor of Milan wanted to see him, and at Cardano’s house, according to Tartaglia, on March 25, 1539, the latter said, *“I swear to you by the Holy Gospels and as a true nobleman never to publish these discoveries of yours if you teach them to me.”* Tartaglia then told him the way to solve all three cubic equations in the form of a poem. And Tartaglia warned Cardano: *“If you do not keep the word of honor you have given me, I promise you to print a book immediately afterwards that will not be very pleasant to you.”*

But the Solution is not complete yet

Tartaglia could now have published his discovery. But he did not do so because he had no solution for the remaining ten cubic equations with a quadratic member, nor did he know what to do in the case of the (later called) *casus irreducibilis*, namely the case where square roots of negative numbers appear in the solution formula.

Cardano and Ferrari

In 1539 and 1545 a book Cardano published under the title *Artis magnae sive de Regulis algebraicis Liber unus*, in which he published the solutions of cubic equations without a square member as the discovery of Scipione dal Ferro, but in two places he also gave Nicolaus Tartalea as the second discoverer. In this algebra book, Cardano showed how to transform cubic equations with a quadratic member into those with a linear member, and thereby lead them to a solution, which Tartaglia never succeeded in doing. That is, in this work one finds the instructions for solving all 13 cubic equations and also the 4th degree equations discovered by Cardano’s student Lodovico Ferrari (1522-1565).^[4]

A contest on geometry, arithmetic, and all disciplines dependent on them

Tartaglia was seething with anger at Cardano’s betrayal. And he wrote the *Quesiti* in 1546 also to vilify Cardano in Task LX as doltish, endowed with little intelligence and reason, trembling in fear of a second-rate arithmetician, a poor sap and incapable of solving easy problems. Lodovico Ferrari then stepped up to defend his former teacher. On February 10, 1547, he addressed the first pamphlet (Italian: *cartello*) as a challenge to Tartaglia and sent it to numerous prominent Italian figures, whom he lists at the end of the twelve-page pamphlet. Ferrari, then 25 years old, challenged Tartaglia to a contest on geometry, arithmetic, and all disciplines dependent on them.

Cartelli and Riposte

The two opponents exchanged six *cartelli* and six *risposte* (answers). The last one is dated July 24, 1548 by Tartaglia, who was already in Brescia at that time. In the second answer Tartaglia gives 31 tasks, in the third *Cartello* Ferrari as many. Both later declared that the opponent had not solved them or had not solved them correctly. Tartaglia taught Euclid in Brescia from March to the end of July 1548. When the hearers went to the country for the harvest, he decided to stop exchanging pamphlets with Ferrari and go to Milan for a public argument with Cardano

and Ferrari. But Cardano, who had already stayed out of the discussion, left Milan, and so only Tartaglia and the brilliant mathematics lecturer Ferrari faced each other on August 10, 1548, in the church of Santa Maria del Giardino, located near the future Teatro alla Scala opera house. The majority of the audience was on Ferrari's side, but this was not the only reason why Tartaglia lost out.

Agonizing Laborious Inventions

In May 1551, Tartaglia published a book of only 38 pages, the *General Rule for using reason and measure to lift not only any sunken ship but also a solid metal tower, called Travagliata Inventione* (agonizing, laborious invention). At the same time, discussions of Nicolo Tartaglia about his *Travagliata Inventione*, a book of 48 pages, appeared. In the *Third Discussion* is told the reason to have titled his invention agonizing invention. “*I chose the title because I was under the greatest sufferings and agonies of my life when I found the main subject of this invention*” and then Tartaglia describes in 13 pages how he was cheated of his agreed payment during his Euclid lectures in Brescia in 1548/49.

In the last years of his life in Venice, Tartaglia wrote a great work on arithmetic, geometry and algebra, but only up to quadratic equations and without a word on cubic ones, the *General trattato di numeri et misure* (*General Treatise of Numbers and Measures*) in six parts, with many remarkable details – the best encyclopedia of mathematics of his time. The beginning appeared in 1556 while Tartaglia was still alive. The last parts came out posthumously in 1560.

References and Further Reading:

- [1] [Archimedes lifted the world off its Hinges](#), SciHi Blog
- [2] [Euclid of Alexandria – the Father of Geometry](#), SciHi Blog
- [3] [Gerolamo Cardano and Physician, Mathematician, and Gambler](#), SciHi Blog
- [4] [Lodovico Ferrari and the quartic equations](#), SciHi Blog
- [5] Masotti, Arnaldo, *Niccolò Tartaglia* in the *Dictionary of Scientific Biography*.
- [6] Feldmann, Richard W. (1961). “[The Cardano-Tartaglia dispute](#)”. *The Mathematics Teacher*. **54** (3): 160–163.
- [7] Chisholm, Hugh, ed. (1911). “[Tartaglia, Niccolò](#)”. *Encyclopædia Britannica*. Vol. 26 (11th ed.). Cambridge University Press.
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- [9] Tartaglia, Niccolò (1543). [Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi](#). Venice.
- [10] Tartaglia, Niccolò (1543). [Euclide Megarense philosopho](#). Venice.
- [11] Tartaglia, Niccolò (1556–1560), *General Trattato di Numeri et Misure*, Venice: Curtio Troiano.
- [12] O'Connor, John J.; Robertson, Edmund F., “[Niccolò Fontana Tartaglia](#)”, *MacTutor History of Mathematics archive*, University of St Andrews
- [13] [500 years of NOT teaching THE CUBIC FORMULA. What is it they think you can't handle?](#), [Mathologer](#) @ youtube
- [14] [Timeline of 16th Century Italian mathematicians](#), via DBpedia and Wikidata