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Hermann Weyl – between Pure Mathematics and Theoretical Physics

Hermann Weyl (1885-1955), photo: ETH Zürich – ETH-Bibliothek Zürich, Bildarchiv [CC-BY-SA 3.0]

On November 9, 1885, German mathematician, theoretical physicist and philosopher **Hermann Weyl** was born. Weyl was one of the most influential mathematicians of the 20th century. His widely varied contributions in mathematics linked pure mathematics and theoretical physics. He made significant contributions to quantum mechanics and the theory of relativity. He attempted to incorporate electromagnetism into the geometric formalism of general relativity.

"In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain."
– Hermann Weyl (1939) [11]

Hermann Weyl – Early Years

Hermann Weyl was born in Elmshorn near Hamburg, Germany and already in his early years his enthusiasm for mathematics and science in general became obvious. Weyl attended the Gymnasium Christianeum in Altona. On the recommendation of the principal, who was a cousin of David Hilbert [4] and who was impressed by the boy's talent, Weyl began to study mathematics with Hilbert in Göttingen after his Abitur in 1904, and also physics on the side. He also took courses in philosophy with Edmund Husserl,[5] where he met his future wife Helene. Except for one year (1905) in Munich, he studied in Göttingen, where he received his doctorate in 1908 under David Hilbert with his thesis "*Singuläre Integralgleichungen mit besonder Berücksichtigung des Fourierschen Integraltheorems" (Singular Integral Equations with Special Consideration of Fourier's Integral Theorem).* In his thesis, Weyl investigated singular integral equations and looked in depth at Fourier integral theorems. Weyl then habilitated in 1910 and taught as a Privatdozent until 1913.

Topology

In Göttingen, Weyl managed to earn himself a great reputation in mathematics and physics. His course on Riemann surfaces resulted in his first book *Die Idee der Riemannschen Fläche* which was published in 1913, in which the previously rather heuristically introduced topological methods were treated more rigorously and the modern concept of manifolds was used systematically for the first time. Weyl's impressive work united analysis, geometry and topology, making rigorous the geometric function theory developed by Riemann. While still teaching in Göttingen, Weyl was also influenced by the philosopher Edmund Husserl.[5] One of Husserl's students, Helene Joseph from Ribnitz, became Weyl's wife and both increased also Weyl's interest in philosophy. Riebnitz later translated many works by the Spanish philosopher José Ortega y Gasset.

Space – Time – Matter

In 1913, Hermann Weyl left Göttingen for Zurich where he held the chair of mathematics and he was probably a colleague of Albert Einstein [7] who was at the time working on his theory of general relativity. Since his book on general relativity, Weyl was strongly interested in connections to physics. He formulated the underlying differential geometry in a more general and flexible way introducing an affine connection. Also Einstein counts as one of the key influencers of Weyl's work along Erwin Schrödinger who was also appointed professor at ETH Zurich back then. Weyl's 1917 lecture on relativity through differential geometry became the foundation for his second book titled *Raum-Zeit-Materie (Space – Time – Matter)*., in which he introduced for the first time the concept of a gauge theory, but at first not in the present form, but by a locally variable scale factor. The idea behind this was that when a vector is transported in parallel along a closed curve, not only the direction is changed (which is expressed by the curvature), but also the length could be variable. In this way, he hoped to incorporate electrodynamics into the theory. The book was published in 1918 with further editions in later years.

The Theory of Lie Groups

His most important work (*Mathematische Zeitschrift*, Volumes 23/24, 1925/1926) is perhaps in the theory of Lie groups, whose representation theory he explored, introducing global concepts such as manifolds, rather than the local aspects of Lie algebra which had predominated until then. For example, he explained spinors for the first time from the topology of the spin group. He also proposes here a connection with the methods of the representation theory of finite matrix groups developed by Ferdinand Georg Frobenius [8] and Issai Schur. In the 1920s and throughout the 1930s, Hermann Weyl worked on the concept of continuous groups using matrix representations. His theory of representations of semisimple groups is now considered one of Weyl's greatest achievements.

Later Years

He turned down a call to Göttingen to succeed Felix Klein.[9] Only in 1930, after Hilbert's chair had been vacated, did he accept: Becoming Hilbert's successor was an honor he could not refuse. However, the move from Zurich to Göttingen was not easy for him, as he viewed with concern the political radicalization and rise of National Socialism in the Weimar Republic, as he expressed in a 1930 address to the Göttingen Mathematical Association: "*It is only with some trepidation that I find myself returning from its [traditionally democratic Switzerland's] freer and more relaxed atmosphere to the yawning, gloomy and tense Germany of the present.*" Throughout his life he felt committed to democratic ideals, and in 1933 he found himself unable to teach in Nazi-ruled Germany, especially since his wife was Jewish. In his letter of resignation sent from Zurich on October 9, 1933, to the new National Socialist Minister of Education, Bernhard Rust, he wrote: "*That I am out of place in Göttingen became clear to me very soon when I moved there in the fall of 1930 as Hilbert's successor after 17 years at the Swiss Federal Institute of Technology in Zurich.*" Through the mediation of Albert Einstein, he accepted a position at the Institute for Advanced Study in Princeton, where he worked until 1951. In Princeton his wife Helene died in 1948, and in 1950 he married the sculptor Ellen Bär, daughter of Richard Bär of Zurich, who made the Hermann Weyl bust that stands in his memory at the universities of Princeton, Zurich, and Kiel. He spent the last years of his life mainly in Zurich. In 1955 he was made an honorary citizen of his native town of Elmshorn, and shortly thereafter he died unexpectedly on December 8, 1955, in Zurich due to a heart attack he suffered while sending mail to a mailbox.

References and Further Reading:

- [1] O'Connor, John J.; Robertson, Edmund F., "Hermann Weyl", *MacTutor History of Mathematics archive*, University of St Andrews
- [2] Hermann Weyl at the National Academy of Sciences
- [3] Hermann Weyl at zbMATH
- [4] David Hilbert's 23 Fundamental Problems, SciHi Blog
- [5] Edmund Husserl's Phenomenology, SciHi Blog
- [6] Bernhard Riemann's innovative approaches to Geometry, SciHi Blog

- [7] How Albert Einstein Revolutionized Physics, SciHi Blog
- [8] Ferdinand Georg Frobenius and Group Theory, SciHi Blog
- [9] Felix Klein and the Klein-Bottle, SciHi Blog
- [10] Sir Michael Atiyah, *Beauty and Truth in Mathematics; a Tribute to Albert Einstein and Hermann Weyl*, Institute for Advanced Study, November 8, 2010, Institute for Advanced Study@ youtube
- [11] Weyl, Hermann (1939b), “Invariants”, *Duke Mathematical Journal*, **5** (3): 489–502, doi:10.1215/S0012-7094-39-00540-5
- [12] 1918. *Raum, Zeit, Materie*. 5 edns. to 1922 ed. with notes by Jūrgen Ehlers, 1980. trans. 4th edn. Henry Brose, 1922 *Space Time Matter*, Methuen, rept. 1952 Dover
- [13] Weyl, H. (1944). “David Hilbert. 1862-1943”. *Obituary Notices of Fellows of the Royal Society*. **4** (13): 547–553.
- [14] Hermann Weyl at the Mathematics Genealogy Project
- [15] Hermann Weyl at Wikidata
- [16] Timeline for Hermann Weyl, via Wikidata