PIERRE DE FERMAT (1607/1608 – January 12, 1665)

by HEINZ KLAUS STRICK, Germany

In 2001, the French postal service released the stamp pictured above on the occasion of the 400th birthday of PIERRE DE FERMAT. However, his precise birthdate cannot be verified. To be sure, there is an entry in the baptismal register in Beaumont-de-Lomagne (near Toulouse) for the date August 20, 1601, attesting to the baptism of one PIERRE FERMAT. Yet according to the inscription on his tombstone, FERMAT died on January 12, 1665, at the age of 57, implying that he was born in 1607 or 1608.

The well-to-do leather merchant DOMINIQUE FERMAT was married to FRANÇOISE CAZENEUVE, who in 1601 gave birth to a child named PIERRE. After the death of FRANÇOISE, DOMINIQUE married CLAIRE DE LONG. One may conjecture that the boy PIERRE died young and that the child born to the second marriage was given the same Christian name as his half brother.

After attending the local school run by the Franciscans, PIERRE FERMAT attended university in Toulouse and in Bordeaux, where he showed great interest in mathematical subjects. In Orléans, he completed his legal studies, and in 1631 was admitted to the bar in Toulouse.

Named a *conseiller au Parlement* (the Parlement included judicial functions), he dealt with citizens' petitions to the government in Paris. By virtue of his official position, he was entitled to call himself *DE FERMAT*. Over the years, he occupied a number of posts at the high court in Toulouse. His professional standing provided him with a secure income. An internal court document describes the jurist FERMAT as well educated but distracted and preoccupied. That he continued to rise in the bureaucracy can be attributed to his incorruptibility and the fact that many jurists of the court perished during an epidemic of plague.

What caused FERMAT's distraction and preoccupation was mathematics.

Already as a university student he attempted to reconstruct the lost work *De locis planis* of APOLLONIUS OF PERGA (ca. 260–190 b.c.e.) from isolated references and quotations. In his article *Ad locos planos et solidos isagoge* he introduced a number of important ideas about analytic geometry – before DESCARTES. Following through some ideas of FRANÇOIS VIÈTE (1540–1603), he employed algebraic techniques to solve geometric problems. He described curves in the plane using equations in two variables in a coordinate system, in particular the conic sections (circle, ellipse, parabola, hyperbola) using equations of the second degree.
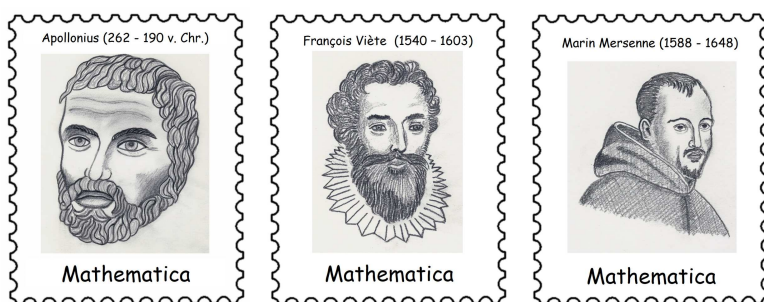
(drawings by Andreas Strick)

In 1636, FERMAT began a correspondence with a circle of Parisian mathematicians around the monk MARIN MERSENNE (1588–1648), to whom he posed a series of problems to which he had already discovered the solutions. Despite repeated requests, he never took the time to work out the details of the methods that he had developed.

Many found FERMAT's work *Methodus ad disquirendam maximam et minimam* on determining the tangents to a curve, extreme values, and surface areas under the curve of a power function (the FERMAT *parabola* $y = x^n$ and the FERMAT *hyperbola* $y = \dfrac{1}{x^n}$) to be extremely opaque. However, ISAAC NEWTON (1643–1727) called it a source of inspiration, and PIERRE-SIMON LAPLACE (1749–1827) – in the spirit of nationalistic enthusiasm – saw in FERMAT the true inventor of the calculus.

The extreme value problems solved by FERMAT include the following:

- At what point in the interior of a triangle with internal angles less than 120° is the sum of the distances to the three vertices minimal?
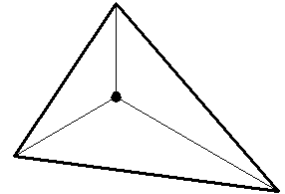
For this FERMAT *point*, the following holds: The line segments joining this point to the three vertices always form angles of 120°.

FERMAT criticized RENÉ DESCARTES's work on optics as flawed, which provoked an angry response from DESCARTES, who in FERMAT recognized a rival of at least equal stature. Twenty years later, FERMAT again attacked the problem of the refraction of light and derived a fundamental law of optics that describes the path of a light ray as it passes from one medium to another: Light takes the fastest, not the shortest, route between points *A* and *B* (FERMAT's principle) – according to SNELL's law of refraction of light $\dfrac{v_{air}}{v_{glass}} = \dfrac{\sin(\alpha)}{\sin(\beta)}$.
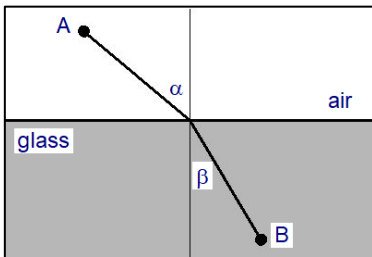
From 1643 to 1654, FERMAT lost contact with his mathematical colleagues in Paris due to civil war and an epidemic of plague. Inspired by the *Arithmetica* of DIOPHANTUS OF ALEXANDRIA (ca. 250 b.c.e.), he immersed himself in a branch of mathematics that had been largely neglected by the mathematicians of his time: number theory.

Five years after FERMAT's death, his son, CLÉMENT-SAMUEL, discovered in the margin of his father's copy of the *Arithmetica* (a Latin translation with commentaries published by BACHET DE MÉZIRIAC, 1581–1638) an assertion that became known as FERMAT's *last theorem*:

- The Diophantine equation $x^n + y^n = z^n$ has no nontrivial solutions in integers *x*, *y*, *z* for $n > 2$.

Instead of stating an idea of a proof, FERMAT wrote down the following famous sentence: *Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.*
(I have found a truly wonderful proof, but this margin is too small to contain it.)

One must assume that FERMAT was mistaken. Over the centuries, many mathematicians searched for a proof, but it was not until 1995 that ANDREW WILES published a proof, which required much of the machinery, unavailable to FERMAT, of modern mathematics.

FERMAT never again referred to this theorem in its full generality, which suggests, perhaps, that he had become aware of his error.

He did give a proof of the theorem for the special case *n* = 4, using the method of infinite descent (*descente infinie*) that he had developed: On the assumption that there is a solution (*x*, *y*, *z*) to the equation $x^4 + y^4 = z^4$ in natural numbers *x*, *y*, and *z*, FERMAT produced, using his method, a further solution (x_1, y_1, z_1) with $x_1 < x$, $y_1 < y$, $z_1 < z$. And by repetition of the method, he then produced an infinite series of such solutions, which is impossible because the natural numbers are bounded from below.

In 1640, BLAISE PASCAL wrote to FERMAT asking for confirmation of some of his own ideas on the solution to two problems that had been posed to him by ANTOINE GOMBAUD, CHEVALIER DE MÉRÉ:

- Why is it advantageous to wager that on the throw of four dice, at least one will turn up a six, but not that in 24 throws of two dice, at least one double-six will be rolled? (*problème des dés*).

- In a game of chance involving two players competing in a number of rounds, the winner of the entire amount staked is the one who is the first to achieve a specific number of points. At some moment, the game has to be adjourned. What is a fair division of the stakes? (*problème des partis*).

The resulting exchange of letters is considered the birth of probability theory.

FERMAT attempted in vain to interest PASCAL in problems in number theory. He posed a host of such problems to his correspondents throughout Europe. For example:

- Find all integer solutions of the equation $Nx^2 + 1 = y^2$, $N \in \mathbb{N}$.

- Prove that the equation $x^2 + 4 = y^3$ has precisely two solutions, while the equation $x^2 + 2 = y^3$ has just one.

He discovered that prime numbers of the form $4n + 1$ can be uniquely expressed as the sum of two squares ($5 = 2^2 + 1^2$; $13 = 3^2 + 2^2$; $17 = 4^2 + 1^2$; $29 = 5^2 + 2^2$; and so on), while no prime number of the form $4n - 1$ is the sum of two squares.

If p is a prime number and a is an integer that is not divisible by p, then the number $a^{p-1} - 1$ is always divisible by p. FERMAT used this fact, which today goes by the name *FERMAT's little theorem*, as a test for primality.

His conjecture that all integers of the form $p_n = 2^{2^n} + 1$, that is, $p_0 = 2^{2^0} + 1 = 3$, $p_1 = 2^{2^1} + 1 = 5$, $p_2 = 2^{2^2} + 1 = 17$, $p_3 = 2^{2^3} + 1 = 257$, $p_4 = 2^{2^4} + 1 = 65.547$, . . . , are prime (so-called *FERMAT primes*) turned out to be false, as discovered by LEONHARD EULER in 1732:
$p_5 = 2^{2^5} + 1 = 4.294.967.297 = 641 \cdot 6.700.417$.

In 1643, FERMAT developed an ingenious procedure for the factorization of large numbers; in a letter to MERSENNE, he exhibited it using the number $n = 2.027.651.281$. The algorithm begins with the smallest integer x that is greater than $\sqrt{n}$. If $x^2 - n$ is a perfect square y^2, then $n = (x - y) \cdot (x + y)$ is a factorization. Otherwise, one checks this result for the number $x + 1$.

Since FERMAT wrote only a few works with a fully reasoned exposition, choosing instead to promulgate his ideas in his vast correspondence, it was only many years after his death that the world recognized just how great a mathematician he was.

Here an important hint for philatelists who also like individual (not officially issued) stamps:

Enquiries at europablocks@web.de with the note: "Mathstamps"