LUCA PACIOLI (1445–1517)

by HEINZ KLAUS STRICK, Germany

The Italian postage stamp from the year 1994 illustrated here recalls the first textbook of mathematics written in the Italian language. It was published in 1494, the work of LUCA PACIOLI: *Summa de arithmetica, geometria, proportioni et proportionalita* (summary of arithmetic, geometry, and algebra). The portrait shows the scholar in the act of explaining a theorem of geometry; on the table before him can be seen a regular dodecahedron.

LUCA PACIOLI grew up in San Sepolcro, a town about 100 kilometres to the southeast of Florence. He may have spent a considerable amount of time at the studio of PIERO DELLA FRANCESCA, for it turned out later that he was very familiar with the artist's work.

He then entered the service of a wealthy Venetian merchant, taking responsibility for some of the business, but primarily for the education of the merchant's three sons. At the same time, he increased his knowledge of mathematics. At the age of 25, he wrote his first book on arithmetic. When the merchant died, he moved to Rome to work for LEONE BATTISTA ALBERTI, advisor to the pope and a philosopher, mathematician, architect, and influential builder. Under ALBERTI's influence, PACIOLI studied theology and entered the Franciscan order.

The year 1477 saw the beginning of a turbulent phase in PACIOLI's life. He taught at a number of universities, in Perugia, Zara (today Zadar in Croatia, then belonging to Venice), Naples, and Rome. The frequent change of location was a result of wars between the various city-states. He wrote two further books on arithmetic for his students. In 1489, he returned to his home city of San Sepolcro, the holder of papal privileges. However, jealousies among members of other religious orders resulted in his being unable to continue his profession as a teacher.

Over the next several years, PACIOLI worked on his *Summa*, which finally appeared in 1494 in Venice. This work contained few new ideas; PACIOLI's particular contribution was that he was able to collect and expound the entire mathematical knowledge of his time in a work of six hundred closely printed pages. The book appeared in the Italian language (that is, not in the usual Latin), which led to its being widely disseminated. And moreover, as a printed book, it could be reissued, and indeed it was. Later mathematicians frequently referred to the *Summa*, and to that extent, LUCA PACIOLI may be said to have influenced the further development of mathematics considerably. One might even say that the publication of his book ushered in a new flowering of mathematics in Italy.

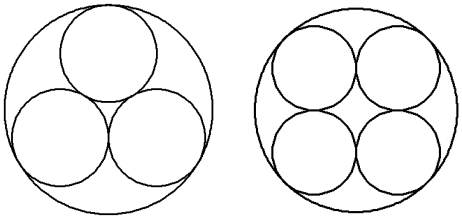
By the end of the fifteenth century, the Arabic-Indic numbers had largely replaced Roman numerals in Europe. However, sums and differences were not written as they are today. PACIOLI proposed writing *p* as shorthand for *plus* (italian: *più* = more) and *m* for *minus* (Italian: *meno*). Moreover, he used the symbol *R* for the square root (*radix*). It was still a complicated matter to express relationships between quantities.

For example, the equation $x^4 + x = x^2 + a$ would be expressed thus: *Censo de censo e cosa equale a censo e numero*, where *cosa* represented the unknown number, *censo* the square of this number, and *censo de censo* for the fourth power.

The *Summa* comprised eight chapters. The first contains a summary of the books of EUCLID on fundamental geometric constructions, calculations of areas, and similarity theory.
The second is concerned with special lines in a triangle.
The third treats right triangles and the associated solution of quadratic equations (theorem of PYTHAGORAS). In contrast to the writings of AL-KHWARIZMI (780–850), here the solution of quadratic equations is presented theoretically, not by means of examples. In addition, PACIOLI dealt with equations of the third and fourth degree, which he held to be generally unsolvable ("*impossibile*"), an assertion that not long afterwards was refuted by SCIPIONE DEL FERRO (1515, but not published), NICCOLÒ TARTAGLIA (1535), and GIROLAMO CARDANO (1545).

The fourth chapter concerns the theory of the circle. Tables of chords give information on the lengths of chords and their associated arcs. For π, PACIOLI gives the approximation $3\frac{33}{229}$. In the fifth chapter, the division of geometric figures is discussed (theory of ratios). The sixth chapter explains how to calculate the surface area and volume of geometric solids. The seventh chapter introduces apparatus and methods of measurement.

The eighth chapter contains a number of applications of different types: calculation of the volume of a barrel (approximately described as two frusta of cones), calculations on regular solids, the inscribing of several equal circles of maximal size in a triangle and in a circle.

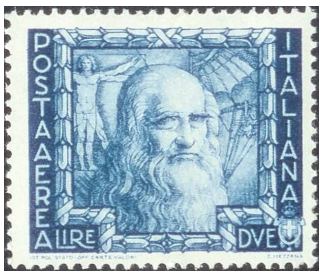


Finally, PACIOLI provides an overview of the coinage and weights and measures of the various Italian city-states.

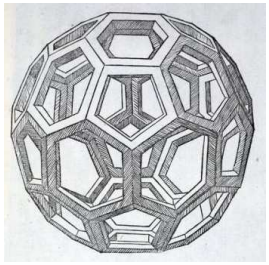
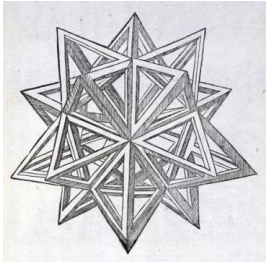
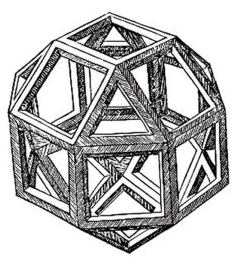
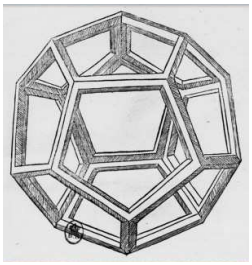
Another postage stamp, which also appeared in 1994, indicates an additional aspect of the content of chapter 8: PACIOLI gives an introduction to the so-called "*Venetian method of bookkeeping*," which realizes the principle of the double-entry accounting system. Although PACIOLI assuredly did not invent the system, he was the first to give a self-contained description of the method, and therefore he is known as the "*father of bookkeeping*."

Finally, the *Summa* contains a problem, later given the name *Problème des partis* by MONTMORT and today known as the *problem of points* or the *problem of division of the stakes*: how are the stakes put up by two players to be fairly divided if a game consisting of a number of rounds that each player has an equal chance of winning has to be interrupted after several rounds and cannot be continued? PACIOLI gives as answer the quotient of the number of rounds won up to the point of interruption and the total number of rounds played, an approach that does not reflect the solution that was eventually developed by PASCAL and FERMAT in their famous correspondence from the year 1654.

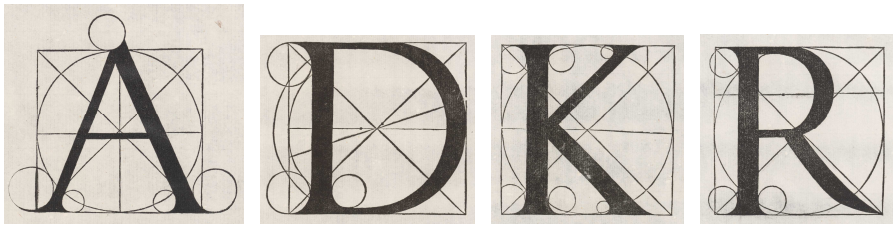
When in 1496, LUDOVICO SFORZA became the new ruler of Milan, he invited PACIOLI to become his court mathematician. There he met LEONARDO DA VINCI, who was in the service of the duke as artist and engineer, and the two became friends.



PACIOLI now began work on a new project, *Divina proportione* (the golden section), for which LEONARDO DA VINCI executed the drawings. This book, like the *Summa*, was published in Venice, though it did not appear in print until 1509.

On the basis of corresponding sections in EUCLID's Elements, he dealt in the first chapter of the book with the regular and semigregular polyhedra. The second chapter explains the significance of the golden section, particularly in architecture. The third contains a translation of texts written by PIERO DELLA FRANCESCA in Italian.
In the book are also to be found some of PACIOLI's designs for typefaces, which a few years later were picked up by FRANCESO TORNIELLO, a pioneer in mathematical typography (particularly in the design of large initial letters of a text).



After the conquest of Milan by French forces, PACIOLI fled to Venice, then later to Florence, where he taught geometry at the university for several years. After his selection as superior of his order in Romagna in 1506, he entered the monastery of Santa Croce in Florence. In 1509, he moved to Venice, and in the following year to Perugia and then finally in 1514 to Rome, where, now aged 70, he still lectured on mathematics.

It is unknown whether he died in Rome or in San Sepolcro. At his death he left unfinished a collection of mathematical puzzles and tricks: *De viribus quantitatis*.

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https://www.spektrum.de/wissen/luca-pacioli-1445-1517/1009133

Translated by David Kramer

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