**THOMAS BAYES** (1701/2 – April 17, 1761)

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THOMAS BAYES was born the eldest of seven children of the Presbyterian clergyman JOSHUA BAYES and his wife ANNE CARPENTER. No documents exist about the date and place of the boy's birth; it is only known from his gravestone that THOMAS BAYES died at the age of 59.

It is doubtful whether the only existing portrait actually shows THOMAS BAYES. The portrait was first printed in a book that was not published until 1936. In the opinion of experts, however, neither the type of the wig nor the style of the frock nor the shape of the collar fit the time in which THOMAS BAYES lived and worked.

As the son of a nonconformist clergyman, THOMAS presumably received his schooling at one of the Dissenters' academies (see below) near his home in London.

Because of his outstanding knowledge of mathematics, which later became apparent, especially in probability, it has been assumed that he received additional instruction from ABRAM DE MOIVRE. The latter lived as an emigrant in London and had to earn his living mainly by giving private lessons.

In 1719 BAYES enrolled at the University of Edinburgh and attended lectures in theology, logic and mathematics. Two sermons he preached on texts from the Gospel of Matthew as part of his education are documented.

After completing his studies, he returned to London, was ordained and assisted his father in his congregation. During this time he also wrote the theological controversy *Divine Benevolence* on the question *How can the existence of evil be reconciled with the goodness of God?*

In 1731 he became pastor of a Presbyterian congregation in Tunbridge Wells, a spa town about 30 miles southeast of London, popular with the upper classes of Londoners for its iron-rich springs. Reverend BAYES held his office until 1752 though he had already tried in vain to find a successor some years before. He continued to reside in Tunbridge Wells until his death in 1761.

Rather modest and reserved by nature, he shunned publicity – certainly not a favourable prerequisite for his work as a parish priest. He was considered an excellent scholar, but not necessarily a gifted preacher. His theological views tended towards Arianism, i.e. he rejected the doctrine of the Trinity (as, incidentally, did ISAAC NEWTON). It is doubtful whether all the members of the congregation agreed with his theological views.

From today's point of view, a career as a mathematician or natural scientist would have been more suited to BAYES' interests and abilities.

In 1734, when the Irish theologian and philosopher GEORGE BERKELEY published *The Analyst – A Discourse Addressed to an Infidel Mathematician: Wherein It Is Examined Whether the Object, Principles, and Inferences of the Modern Analysis Are More Distinctly Conceived, or More Evidently Deduced, Than Religious Mysteries and Points of Faith*, BAYES was able to demonstrate his mathematical education.

BERKELEY had attacked all non-believers (including the non-conformists) in his work and had severely criticised NEWTON's differential and integral calculus: it produced useful results but lacked logical foundations.
Hardly any other contemporary than BAYES would have been able to write a comparably competent contribution to the defence of NEWTON's doctrine of fluxions (An Introduction to the Doctrine of Fluxions, and a Defence of the Mathematicians Against the Objections of the Author of The Analyst). Although BAYES had published his contribution anonymously, his authorship was not concealed and in 1742, "the highly educated gentleman, whose merits are well known", was admitted to the Royal Society – albeit without explicit reference to this writing.

When BAYES died unmarried and childless, he left his extensive library to his successor in office. His considerable fortune (which he himself had inherited from his father, and he in turn from rich ancestors) he bequeathed to his relatives and friends. He had decreed that the funeral expenses should be kept as low as possible and a funeral oration should be dispensed with.

RICHARD PRICE, one of his best friends, like BAYES the pastor of a nonconformist congregation, and later one of the most important personalities of the newly founded United States of America, discovered in his estate, among other things, a notebook in which BAYES had recorded – mostly in shorthand – his reflections on mathematical and physical questions over many years. For example, he found instructions on how to calculate the time and place of the conjunction of two planets, and descriptions of experiments with electric charges.

In addition, PRICE discovered the manuscript of an essay that he submitted in 1763 – with his own comments and a preface – for publication in the Philosophical Transactions of the Royal Society:

Dear Sir, I now send you an essay which I have found among the papers of our deceased friend Mr. Bayes, and which, in my opinion, has great merit, and well deserves to be preserved.

- An Essay towards solving a problem in the doctrine of chances.

Today we associate the name of THOMAS BAYES with the BAYES theorem named after him. However, we look in vain for a formulation of the theorem as we know it today.

After introductory definitions and first rules, which are also contained in ABRAHAM DE MOIVRE's The doctrine of chances (2nd edition of 1738), one finds as Theorem 5 the description of the procedure how one can conclude from the probability of an event that has occurred to the probability of the circumstances:

- If, of two related events, the probability of the second is \( \frac{b}{N} \) and the probability of both together is \( \frac{P}{N} \), and we have discovered that the second event has actually occurred, then for the correctness of the assumption that the first event has also occurred, \( \frac{P}{b} \) is the probability – in today's notation \( P_b(A) = \frac{P(A) \cdot P_b(B)}{P(B)} \).
Another form of Bayes' theorem can be traced back to a writing by Pierre-Simon Laplace in 1774, which he wrote without knowledge of Bayes' treatise.

In this Mémoire sur la Probabilité des causes par les évènements it is said:

- If an event can be caused by a number $n$ of different causes, then the probabilities of these causes behave like the probabilities of the event from these causes, and the probability of the existence of each of them is equal to the probability of the event from this cause, divided by the sum of all the probabilities of the event from each of these causes – in today’s notation

$$P_B(A_j) = \frac{P(A_i) \cdot P_{A_i}(B)}{\sum_{j=1}^{n} P(A_j) \cdot P_{A_j}(B)}.$$ 

In a second part of his writing, Bayes developed a thought experiment:

A white ball is thrown onto a square plate and randomly remains in one place. The position of this ball (distance to the right and to the left edge) is narrowed down by a sequence of experiments: Further balls are thrown at random, and each time the information is obtained as to whether the ball is left or right of the white ball. From this, a better estimate for the actual position of the white ball is obtained from step to step.

From this thought experiment Bayes developed the following theorem, which Richard Price called Theorem 10:

- By $p$ we denote the probability that a certain event $A$ will occur. In $n$ trials this event has occurred $a$ times and not occurred $b$ times. Then the probability that $p$ lies between two values $x_1$ and $x_2$ is given by

$$\int_{x_1}^{x_2} x^a \cdot (1-x)^b \, dx$$

(in today’s notation).

In an appendix, Price then addressed a problem that the Scottish philosopher David Hume had raised in his An Enquiry Concerning Human Understanding in 1748:

- How certain can we be that the sun will rise in the morning?

The sceptic Hume had disputed that it is logically permissible to conclude a generally valid rule from individual observations (inductive conclusion from the past to the future). Although man tends to assume causality of the observed and expects by habit that future events will at least be similar to those observed so far (the future will resemble the past), this is not justified.

Price took up the example and considered a creature that is new on this earth and observes the sunrise: His expectation of seeing the sun again would increase from day to day, but no finite number of returns would be sufficient to produce absolute or physical certainty.
The *Book of Common Prayer*, introduced in 1549 under Henry VIII, played an important role in England's chequered history. It contains important regulations on the beliefs, rites and prayer texts of the Anglican Church.

During the short period under Mary Stuart it was forbidden but under Elizabeth I it became obligatory again. During the Civil War (1641-49) and the Commonwealth (1650-60), congregations were free to make their own arrangements.

When the monarchy was re-established in 1661 and the *Book of Common Prayer* was to become compulsory again, over 1000 clergy ("Nonconformists") across the country protested against this change. They were stripped of their official posts, but in most cases they found support from the local nobility.

After the *Glorious Revolution* in 1688/89, these dissenters (including Presbyterians, Congregationalists, Baptists and Quakers) were given the right to practise their religion freely with their own registered congregations by the *Act of Toleration* of the new rulers William III and Mary – in contrast to the Catholics, who continued to be persecuted.

However, members of the Nonconformist churches were not allowed to hold public office or study at English universities such as Oxford or Cambridge. This led to the establishment of separate academies for the training of clergy; many went to Leiden (in South Holland) or to Scottish universities to study.