EUGENIO BELTRAMI (November 16, 1835 – February 18, 1900)

by HEINZ KLAUS STRICK, Germany

At the time EUGENIO BELTRAMI was born, Lombardy and BELTRAMI's hometown of Cremona were still part of the Habsburg Empire. Both parents, GIOVANNI BELTRAMI and the Venetian ELISA BAROZZI, came from distinguished artistic families; EUGENIO's father earned his living by selling miniatures.

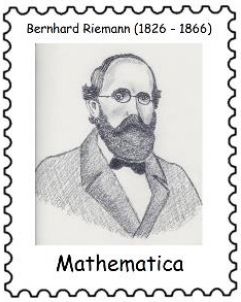
After graduating from school in 1853, EUGENIO enrolled as a student at the Faculty of Mathematics at the University of Pavia, 70 km away.

Thanks to his excellent results in the entrance exam, he was awarded a scholarship and could live in a boarding school, the *Collegio Ghislieri.* But the following year, he was expelled from the college along with five other scholarship holders for allegedly instigating unrest. This disproportionately high punishment had devastating consequences for the talented student. As his family was unable to provide the financial means, EUGENIO BELTRAMI had to abandon his previously very successful studies at the end of 1856 and take a job in Verona as a low-paid secretary to a railway engineer in the province of Lombardy-Veneto.

However, after only three months he was dismissed by the General Director of the railway *for political reasons*. However, this had no consequences, as the political situation had changed radically at the same time. During the Italian War of Independence (the *Risorgimento*), the whole of Lombardy fell to the Kingdom of Piedmont-Sardinia. BELTRAMI kept his job with the railway engineer and followed him to Milan. There he found time to study mathematics again; his first contributions were accepted for publication by the *Annali di Matematica.*

In the meantime, FRANCESCO BRIOSCHI, BELTRAMI's former mathematics professor from Pavia, had been appointed by CAMILLO BENSO, CONTE DI CAVOUR, the leader of the independence movement, to a commission that was to prepare an educational reform for the future united Italy (GIUSEPPE VERDI was also a member of this committee). In 1862, in his capacity as Secretary of State, BRIOSCHI took the opportunity to appoint BELTRAMI to a position as *Professore straordinario* for Algebra and Analytical Geometry at the University of Bologna, even though he had no formal qualifications.

After this step, all academic positions were open to BELTRAMI. In 1863 he accepted an offer from ENRICO BETTI, the rector of the University of Pisa, to take up a professorship in geodesy. In 1866 he returned to Bologna as a professor of rational mechanics. (*Rational mechanics* is the approach of deriving the laws of mechanics using strictly logical methods; similar to geometry, this is based on certain hypotheses (axioms) that are substantiated by experiments.)



Through FRANCESCO BRIOSCHI and ENRICO BETTI, BELTRAMI had come into contact with a branch of mathematics that would particularly interest him in the years to come: non-Euclidean geometry. In 1858 BRIOSCHI and BETTI had visited Göttingen and became acquainted with BERNHARD RIEMANN's ideas on geometry. After their return they translated his works and gave lectures on his approach.

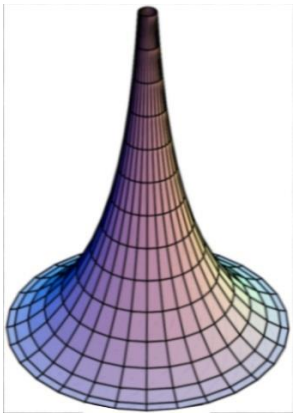
In 1868, BELTRAMI published two articles in the *Annali di Matematica* that attracted international attention: *Saggio d'interpretazione della geometria non-euclidea* (Essay on the Interpretation of Non-Euclidean Geometry) and *Teoria fondamentale degli spazii di curvatura costante* (Fundamental Theory of Spaces of Constant Curvature).

Parallel Axiom and Non-Euclidian Geometry

For centuries, mathematicians had been grappling with the parallel axiom of EUCLID and investigating whether it could be replaced by another one or what the consequences would be if it were replaced by its negation. The greatest progress was made by the Swiss mathematician JOHANN LAMBERT, who was on the verge of discovering hyperbolic geometry in 1766 but did not dare to take the decisive step. CARL FRIEDRICH GAUSS, from whom the term *non-Euclidean* comes, discovered around 1817 that the assumption that the sum of the angles in a triangle is less than 180° leads to a strange but completely consistent geometry. Because he feared that publishing it could damage his reputation, he kept his findings to himself. And when JÁNOS BOLYAI, the son of his friend FARKAS BOLYAI, came to the same conclusions, he did not find the words of appreciation that could have encouraged JÁNOS to continue his research. The publications of NIKOLAI LOBACHEVSKY on an *Imaginary Geometry* initially went unnoticed.

The negation of the parallel axiom has surprising consequences, including:

- In a triangle, the sum of the interior angles is always less than 180°.
- If two triangles have the same angles, then they are congruent.
- The set of all points that have the same positive distance from a given line and lie in the same half-plane of this line do not themselves form a line.
- You cannot always draw a circle through three points that do not lie on a line.



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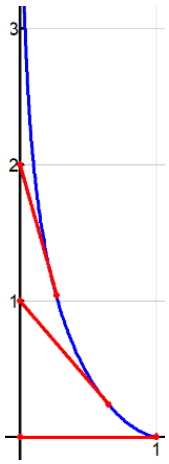
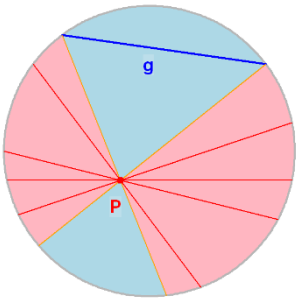
BELTRAMI was the first to be able to provide a model for a hyperbolic geometry.

In his *Saggio* contribution from 1868, he investigated the normal projection of the surface of revolution of a *tractrix* (see right) into the interior of a circular surface.

The name of this curve comes from the Latin word *trahere* (to drag); it describes the movement of the end point of a rod that initially lies (from the origin) on the x-axis and whose starting point is then pulled along the y-axis.



EUGENIO BELTRAMI called this surface with constant negative curvature a *pseudosphere*. Geodesic lines on the pseudosphere become chords in the circular surface through projection, whereby the endpoints do not belong to each of them. Through a point P on the circular surface there are infinitely many chords that have no common point with a given chord g (figure right).



This property corresponds to the negation of the parallel axiom: *for a given straight line g and a point P that does not lie on it, there is more than one parallel through this point* (in this case even infinitely many) .

In the early 1860s, BELTRAMI had met BERNHARD RIEMANN, who was suffering from tuberculosis, during his stay at a spa in northern Italy and exchanged ideas with him. After RIEMANN's death (1866), his friend RICHARD DEDEKIND began to publish the posthumous works, including the text of RIEMANN's unpublished *habilitation* lecture from 1854 (*On the hypotheses underlying geometry*). BELTRAMI, who had not known the content of the lecture until then, was then able to generalise his model in the second paper (*Teoria*). Missing aspects in BELTRAMI's model were later completed by FELIX KLEIN (1871) and DAVID HILBERT (1901).

In 1870, the unification process of Italy was completed and Rome became the new capital of the kingdom. BELTRAMI accepted an appointment at the newly founded University of Rome to a chair in *Rational Mechanics* and was accepted as a member of the newly founded (state) *Reale Accademia dei Lincei*. In 1876, he moved to the University of Pavia to take up a professorship in Mathematical Physics.

The mathematician, who was now much sought after and who now shifted his research work to mathematical applications, considered scientific activity more important than taking on prestigious positions. He finally gave in to pressure from the University of Rome and returned there in 1891. In 1898 he was unanimously elected President of the *Accademia* and in 1899 (after the legally prescribed waiting period) he was also appointed a life member of the *Senato del Regno* (one of the parliamentary chambers) by the Italian king.

The much-honoured mathematician (he was a member of the academies of the universities of Bologna, Milan, Turin, Naples, Paris, Göttingen, Brussels, Munich, Berlin and London) died at the beginning of 1900 at the age of 64.

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