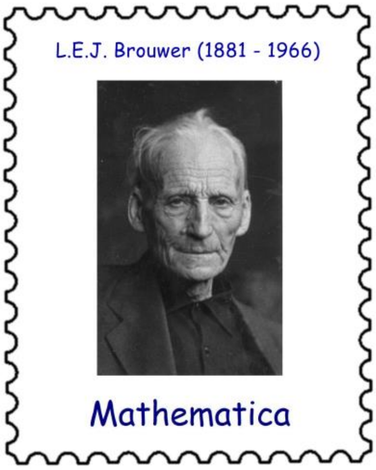
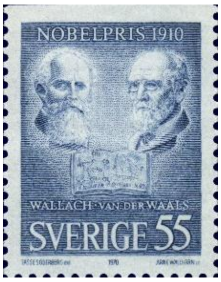
Luitzen EGBERTUS JAN BROUWER
(February 27, 1881 – February 12, 1966)

by HEINZ KLAUS STRICK, Germany

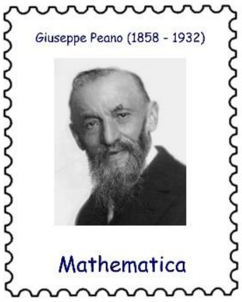


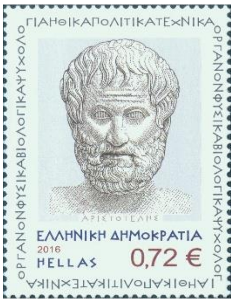
LUIZEN EGBERTUS JAN BROUWER was born as the eldest of three sons of a teacher in Overschie (now a suburb of Rotterdam). He graduated from the *Hogere Burgerschool* in Hoorn with brilliant grades, after which he had to attend the Stedelijk Gymnasium in Haarlem for two years to obtain passes in Latin and Greek – a prerequisite for studying at a university. At the age of 16, he enrolled at the University of Amsterdam to study mathematics and natural sciences.

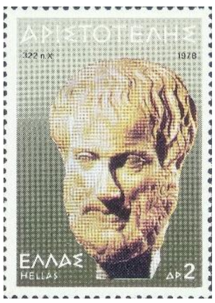


BROUWER attended lectures in physics by the later Nobel 1910 Prize winner JOHANNES DIDERIK VAN DER WAALS and in mathematics by DIDERIK JOHANNES KORTEWEG (who in turn had completed his doctorate under VAN DER WAALS). During his first semester, BROUWER wrote a paper that was accepted and published by the *Royal Netherlands Academy of Sciences.*

A particular influence on BROUWER's future development was the private lecturer GERRIT MANNOURRY, which inspired BROUWER to study the recent publications of GIUSEPPE PEANO and BERTRAND RUSSELL.







The first version of his doctoral thesis *Over de grondslagen der wiskunde* (On the foundations of mathematics, 1907) displeased his supervisor KORTEWEG, as it dealt too much with philosophical questions and too little with "respectable" mathematics, which could also be detrimental to his future academic career as a mathematician. BROUWER followed this advice, but then the following year published the article *De onbetrouwbaarkeit derlogische principes* (The unreliability of logical principles), in which he explicitly rejected the validity of the *principle of the excluded middle* (*principium exclusi tertii*).

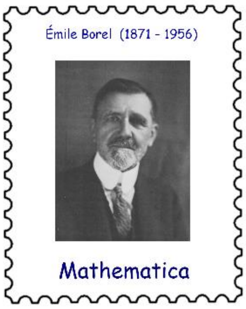
This principle, which goes back to ARISTOTLE, states that for any given statement, *either it is true or the logical* (*complementary*) *opposite is true* – there can be no third possibility ("something in between"). In English, the principle is called *the Law of Excluded Middle* (LEM).

Until the 19th century, it was generally accepted as a "classical" method of reasoning that one can prove the existence of an object by disproving its non-existence. The principle is used, among others, in the method of proof by contradiction. EUCLID had already used this method to prove the existence of an infinite number of prime numbers. This argument is also used in the proof of the theorem that $\sqrt{2}$ is an irrational number and in the proof of the mean value theorem in analysis.

In his critical writings, BROUWER demanded that the body of mathematical theorems must be subjected to a complete revision.

Decades earlier, LEOPOLD KRONECKER called for the *arithmetization of analysis*. His famous statement "*The good Lord made the whole numbers, everything else is the work of man*" at the annual meeting of natural scientists in 1886 showed the path that BROUWER also wanted to take: starting from the natural numbers, which are given by a *primal intuition*, further insights are to be gained through *constructive* procedures, initially in geometry and in the construction of the whole and rational numbers. And only those "elements" that can be *constructed* in a *finite* number of steps should be considered to exist. BROUWER's first publications on the foundations of mathematics met with little response – not least because he had only written these contributions in Dutch.

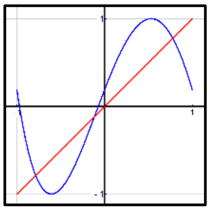
BROUWER concentrated on the investigation of topological questions in geometry, in particular the structure of the so-called LIE groups (named as the fifth of the 23 problems posed by HILBERT in 1900). Today, BROUWER is considered the actual founder of so-called *algebraic topology.* This deals with the algebraic structures of point sets. Before BROUWER, this area was called *analysis situs*. BROUWER's first contributions to this were so impressive that in 1908 he was invited to give a lecture at the International Congress of Mathematicians in Rome – a special honour for the young academic.



After a visit to Paris at the end of 1909, where he discussed topology with HENRI POINCARÉ, JACQUES HADAMARD and ÉMILE BOREL, he succeeded in proving a general *fixed point* theorem that today bears his name (the special cases for *n* = 1 and *n* = 2 had already been proved before):

BROUWER 's fixed point theorem: Every continuous mapping of the *n*-dimensional unit sphere $D^n = \{x \in \mathbf{R}^n | \; \|x\| \leq 1\}$ onto itself has at least one fixed point.

Example (*n* = 1): If *f is* a continuous function with $f:[-1;+1] \to [-1;+1]$, then there exists at least one $a \in [-1;+1]$ with $f(a) = a$. That is, the graph of *f* has at least one intersection with the line $y = x$.

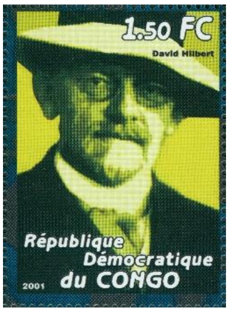


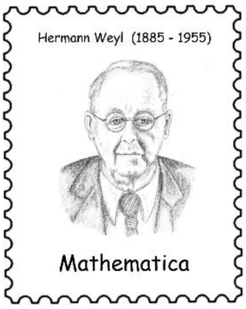
BROUWER himself considered a consequence for the case *n* = 2 to be particularly remarkable: If you dissolve the sugar in a cup of coffee by stirring, then at any time there will be at least one spot on the surface of the liquid that is *not* moving.

After BROUWER had succeeded in proving theorems for arbitrary dimensions (as well as theorems on the invariance of dimension and on the generalization of JORDAN's curve theorem) – all using the indirect proof method – he was again invited to give a lecture at the International Congress of Mathematicians (in Cambridge) in 1912; in the same year he was elected a member of the *Royal Society*.

With the support of DAVID HILBERT, the editor of the *Mathematische Annalen*, in which BROUWER's contributions to topology were published, he was given an extraordinary professorship at the University of Amsterdam, and then a full professorship the following year. He again devoted his inaugural lecture to the fundamental problem.

Hilbert tried in vain to recruit Brouwer for a professorship in Göttingen and Berlin. As early as 1914 he had ensured that Brouwer was included in the Scientific Advisory Board of the *Mathematische Annalen* .





In 1928, however, relations broke down. Against the resistance of the advisory board members Albert Einstein and Constantin Carathéodory, Hilbert forced Brouwer to be excluded from further work on the advisory board. The conflict was triggered by an essay by Hermann Weyl *On the new crisis of foundations in mathematics* from 1921. Weyl was very impressed by Brouwer 's theses. Weyl's teacher Hilbert, however, saw his exclamation " Brouwer – that is the revolution!" as a personal attack ("an *attempted coup* ") on his own efforts to reorganise the logical foundations of mathematics. For Hilbert, the confrontation with Brouwer's *intuitionism* was the reason to resume his own work on his formalistic approach to a contradiction-free structure of mathematics (the Hilbert program). Hilbert compared the abandonment of the indirect proof method to the work of an astronomer whose telescope has been taken away.

Brouwer's intended *constructive* structure of mathematics on the basis of *intuitionistic logic* proved to be extremely complicated and Brouwer found only a few supporters for the approach. Weyl also soon lost interest in the topic.

Brouwer kept his chair until 1951. During his entire career as a university lecturer he never gave a single lecture on topology. Ultimately he considered the theorems for which he had once become famous to be false, since they could not be proved without applying the principle of contradiction.

After the end of World War II, Brouwer was temporarily suspended from his university for alleged collaboration with Nazi Germany, which almost caused him to emigrate. The reason for this: in order to avoid further pressure from the occupying forces, he had advised his students in 1943 to sign a declaration of loyalty to Germany. In fact, Brouwer was active in the Dutch resistance. He employed Jewish staff at his department for as long as possible, including Hans Freudenthal as his assistant (who later published Brouwer's collected works). Even though Brouwer did not succeed in winning over many mathematicians to his teaching of intuitionism, he received numerous honours.

Brouwer died at the age of 85 in an accident while crossing a road in his hometown of Blaricum (province of North Holland).

Bust of Brouwer (with kind permission of *Centrum Wiskunde en Informatica* (CWI) Amsterdam)

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https://www.spektrum.de/wissen/brouwer-und-die-konstruktive-mathematik/2214587

Translated by John O'Connor, University of St Andrews

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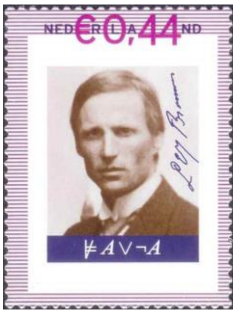

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