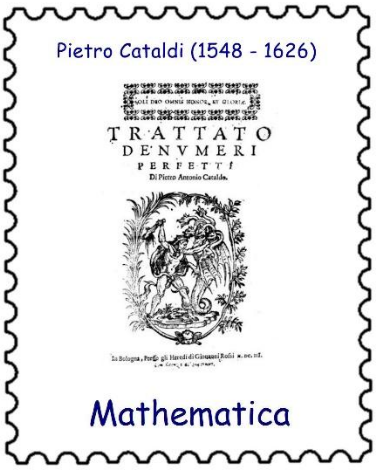
PIETRO CATALDI (April 15, 1552 – November 2, 1626)

by HEINZ KLAUS STRICK, Germany



Little is known about PIETRO ANTONIO CATALDI's early years – his father's first name was PAOLO, and his birthplace Bologna, a city belonging to the Papal States at that time, is also known. It is further recorded that at the age of 17 he was already teaching mathematics at the *Academy of Fine Arts* in Florence and in 1572 he moved to the University of Perugia (Umbria).

In 1584 he returned to his hometown and earned a doctorate in philosophy. Afterwards, he taught mathematics and astronomy at the *Studio di Bologna*, the oldest European university, until his death, and did so in Italian, not in Latin as was customary.

When he died (unmarried) in 1626, according to his last will and testament, a boarding school for poor children was established in his house, and the money he left behind was used to maintain it.

CATALDI wrote over 30 works; the first part of his *Pratica aritmetica, ovvero elementi pratici dell'i numeri aritmetici,* probably at the age of 17. The book was published in printed form in 1602 (under the pseudonym PERITO ANNOTIO), with three further volumes appearing under his real name between 1606 and 1617.

Throughout his active career, he devoted himself intensively to the study of EUCLID's *Elements*. In the last year of his life, he published a three-volume edition OF EUCLID's works (*Difesa di Euclide*).

In 1603 he had come to the conviction that he had succeeded in deriving the parallel postulate from the other four axioms. In his work *Opusculum de lineis rectis aequidistantibus, et non aequidistantibus,* he defined it as follows:

- *A straight line is called equidistant to another straight line* (*in the same plane*) *if two shortest connections from two different points on one line to the other line are always of equal length.*

What he didn't realise was that this definition of the equidistance between two lines is equivalent to the statement of the parallel postulate.

In his work from 1603, CATALDI announced another publication, which then only appeared in 1613: *Trattato del modo brevissimo di trovare la radice quadra delli numeri, et regole da approssimarsi di continuo al vero nelle radici de' numeri non quadrati* (Treatise on the shortest method for finding the square root of numbers and rules for gradually approximating the true value of roots of non-square numbers).

RAPHAEL BOMBELLI (1526-1572), one of the CATALDI's predecessors at the *University of Bologna* had provided an initial proposal for a suitable algorithm:

To determine an approximating fraction for $\sqrt{13}$, he made an estimate based on the next smallest square number and used the approach:

$3+x=\sqrt{13}$, which when squared results in $6x+x^2=4$.

If we neglect x^2, then we are left with $6x \approx 4$, i.e. $x \approx \frac{2}{3}$. From this it follows that $x^2 \approx \frac{2}{3}x$.

From $6x+\frac{2}{3}x=4$ one then obtains the approximate value $x \approx \frac{4}{6+\frac{2}{3}} = \frac{3}{5}$ after which the calculation can be repeated.

CATALDI reformulated the equation $6x + x^2 = 4$ in a different way:

From $x \cdot (6+x) = 4$ one obtains $x = \frac{4}{6+x}$. The number we are looking for x is therefore definitely less than $\frac{4}{6+0} = \frac{2}{3}$. Thus, the following holds: $3 < \sqrt{13} < 3\frac{2}{3}$ with $(3\frac{2}{3})^2 = 13\frac{4}{9}$.

In the next step, he replaced the variable $\frac{4}{6+x}$ in the denominator of the fraction x with the previously obtained fractional term: $x = \frac{4}{6+\frac{4}{6+x}}$. Because $\frac{4}{6+\frac{4}{6+x}} > \frac{4}{6+\frac{4}{6+0}}$ a better approximation for x is now obtained: $\frac{4}{6+\frac{4}{6+x}} > \frac{4}{6+\frac{4}{6+0}} = \frac{4}{\frac{40}{6}} = \frac{3}{5}$ with $3\frac{3}{5} < \sqrt{13} < 3\frac{2}{3}$ and $(3\frac{3}{5})^2 = 12\frac{24}{25}$.

In the next iteration step, i.e., after re-inserting the fractional term, an approximate value is obtained $\frac{4}{6+\frac{4}{6+\frac{4}{6+x}}}$ which is again larger than x, but smaller than $\frac{2}{3}$: $\frac{4}{6+\frac{4}{6+\frac{4}{6+0}}} = \frac{4}{6+\frac{3}{5}} = \frac{20}{33}$ with $(3\frac{20}{33})^2 = 13\frac{4}{1089}$.

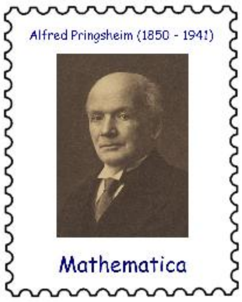
CATALDI realized that by continuing this process, one obtains increasingly better approximations for $\sqrt{13}$:

The next steps result in $\frac{4}{6+\frac{20}{33}} = \frac{66}{109}$, $\frac{4}{6+\frac{66}{109}} = \frac{109}{180}$, $\frac{4}{6+\frac{109}{180}} = \frac{720}{1189}$ etc. with

$$3\frac{3}{5} < 3\frac{66}{109} < 3\frac{720}{1189} < \ldots < \sqrt{13} < \ldots < 3\frac{109}{180} < 3\frac{20}{33} < 3\frac{2}{3}.$$

The fraction $3\frac{720}{1189}$ gives the approximate value of $\sqrt{13}$ with 5-digit accuracy.

For successive approximation fractions $\frac{2}{3}, \frac{3}{5}, \frac{20}{33}, \frac{66}{109}, \frac{109}{180}, \frac{720}{1189}, \ldots$, or in general $\frac{p_1}{q_1}, \frac{p_2}{q_2}, \frac{p_3}{q_3}, \ldots$, the products $p_i \cdot q_{i+1}$ and $p_{i+1} \cdot q_i$ each differ by 1.



Even though CATALDI only demonstrated his calculations using numerical examples, he can rightly be called the discoverer of the approximation method using *continued fractions*.

For the nested fractions, he chose the notation $3 \,\&\, \frac{4}{6.} \,\&\, \frac{4}{6.} \,\&\, \frac{4}{6.}$ where the dot in the denominator indicates where the next fraction should be placed. The notation is reminiscent of the notation commonly used today, introduced by ALFRED PRINGSHEIM: $3 + \frac{4|}{|6} + \frac{4|}{|6} + \frac{4|}{|6} + \ldots$

General representation of CATALDI 's algorithm

From the approach $\sqrt{a^2+b} = a + x$, it follows that by the transformation $x = \frac{b}{2a+x}$ the continued fraction development $\sqrt{a^2+b} = a \,\&\, \frac{b}{2a.} \,\&\, \frac{b}{2a.} \,\&\, \frac{b}{2a.} \,\&\, \ldots$ is obtained.

CATALDI 's name is often mentioned in connection with *perfect numbers*: In 1603 he published his work *Trattato de' numeri perfetti.*

The following theorem has been known since EUCLID:

- If the natural number $n = 2^k - 1$ is a *prime* number, then $m = 2^{k-1} \cdot (2^k - 1)$ is a *perfect* number, i.e., the sum of the proper divisors of this number is equal to the number m.

The prerequisite that the number of type $n = 2^k - 1$ cannot be decomposed into factors is an essential condition.

CATALDI demonstrated in this context:

- *If the natural number* $n = a \cdot b$ *with* $a, b \in \mathbb{N}$ *is a composite number, then* $2^n - 1$ *is also not a prime number; because* $2^{a \cdot b} - 1 = (2^a - 1) \cdot (1 + 2^a + 2^{2a} + 2^{3a} + \ldots + 2^{(b-1) \cdot a})$.

In ancient times, only the four smallest perfect numbers were known ($k = 2, 3, 5, 7$):

(1) $2^1 \cdot (2^2 - 1) = 2 \cdot 3 = 6 = 1 + 2 + 3$,

(2) $2^2 \cdot (2^3 - 1) = 4 \cdot 7 = 28 = 1 + 2 + 4 + 7 + 14$,

(3) $2^4 \cdot (2^5 - 1) = 16 \cdot 31 = 496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248$ and

(4) $2^6 \cdot (2^7 - 1) = 64 \cdot 127 = 8128 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064$

The arithmetic master HUDALRICHUS REGIUS (ULRICH RIEGER) from Freiburg had found the fifth perfect number in 1536:

(5) $2^{12} \cdot (2^{13} - 1) = 33{,}550{,}336$.

CATALDI then discovered the next two in 1603:

(6) $2^{16} \cdot (2^{17} - 1) = 8{,}589{,}869{,}056$ and

(7) $2^{18} \cdot (2^{19} - 1) = 137{,}438{,}691{,}328$

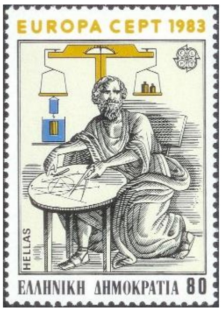
after painstaking calculations using prime number tables.

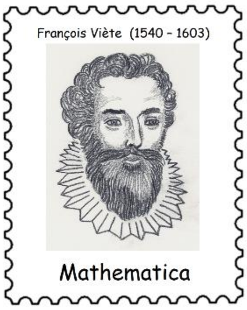
The fourth, fifth, and sixth perfect numbers had already been discovered earlier by the Egyptian mathematician ISMAIL IBN FALLUS (1194–1252), but this was unknown in Europe.

In 1612, CATALDI published his *Trattato della quadratura del cerchio* (Treatise on the Determination of the Area of the Circle) – and with this work he responded to a publication by the French historian and philologist JOSEPH JUSTUS SCALIGER (1540-1609). SCALIGER had made his mark in 1583 with his groundbreaking *Opus de emendatione temporum* (Textbook of Chronology) which gained international renown.

After taking up a professorship in history in Leiden, Netherlands, SCALIGER published his eagerly awaited work *Cyclometrica elementa* in 1594. In this work, he claimed, among other things, that an *inscribed* regular dodecagon has a larger perimeter than the circumference of a circle (!) and that π was equal to $\sqrt{10}$ – a measure that lay outside the interval $3\frac{10}{71} < \pi < 3\frac{1}{7}$ derived by ARCHIMEDES. The non-mathematician SCALIGER dismissed references to calculation errors as irrelevant, since the main point in his arguments was that ARCHIMEDES' method was *logically incorrect*, since he applied the principle of indirect proof (*reductio ad absurdum*) in his derivation.

Besides CATALDI, CHRISTOPHER CLAVIUS and FRANÇOIS VIÈTE, among others, also weighed in, but this did not impress SCALIGER.





LUDOLPH VAN CEULEN, also living in Leiden and a trained fencing master who also taught mathematics, initially did not dare to contradict the statements of the esteemed academic SCALIGER – especially since he himself was not proficient in Latin.

Nevertheless, in 1596 he published a work entitled *Vanden circkel* (On the Circle), in which he examined a regular n-gon with $n = 15 \cdot 2^{31}$ and determined the mathematical constant π to 20 decimal places (since this publication, π has often been referred to as the *Ludolphine number*).

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https://mathematik-ist-schoen.jimdoweb.com/

Translated by John O'Connor, University of St Andrews
