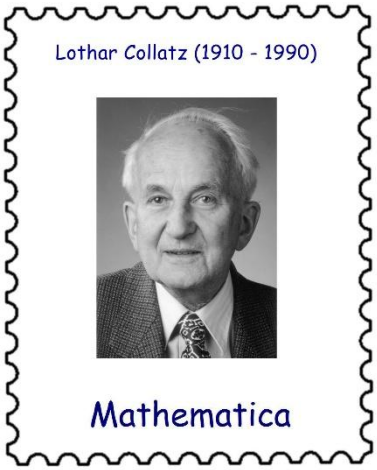
LOTHAR COLLATZ (July 6, 1910 – September 26, 1990)

by HEINZ KLAUS STRICK, Germany



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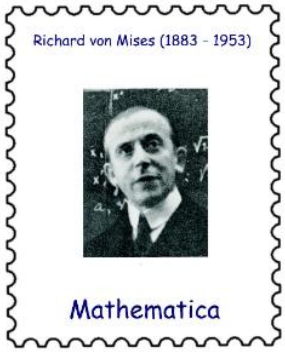
When the National Socialists seized power in the German Reich in January 1933, RICHARD VON MISES, director of the *Institute for Applied Mathematics* at the University of Berlin, felt the threat from the new rulers – despite the exceptions to the law for the restoration of the professional civil service that should have applied to him.

Before he finally gave up his post in Berlin and emigrated to Turkey, he made every effort to find suitable arrangements for his staff.



On 2 November, he sent a message to his student LOTHAR COLLATZ: "*If you still wish to sit the examination with me, you must come at 10 o'clock tomorrow*." He dismissed the student's objection that he was not yet sufficiently prepared. "*That doesn't matter. I have to leave this country. I know you well enough: extraordinary times call for extraordinary measures. I would like to discuss with you at this time how your thesis might develop; for you will now be left to your own devices with your work*."

As it happened, COLLATZ passed his state examination in mathematics and the following day VON MISES left Germany.

LOTHAR COLLATZ was born in Arnsberg in the Sauerland region. Due to his father's work (he was a surveyor), the family first moved to Minden, then to Stettin. His father loved brain teasers, and when he noticed his son's enthusiasm for them, he introduced him to a variety of geometric problems. In Stettin, the boy attended the local grammar school with excellent results.

In 1928 he began studying mathematics and physics in Greifswald, changing his place of study – as was common at the time – to Munich, Göttingen, and Berlin. Influenced by EDMUND LANDAU, ISSAI SCHUR, and OSKAR PERRON, he saw his future in pure mathematics, particularly algebraic number theory. After his sixth semester, however, lectures by RICHARD COURANT and RICHARD VON MISES impressed him so much that he found his focus in applied mathematics.

Throughout his life COLLATZ did not consider it appropriate to emphasise the distinction between pure and applied mathematics as a dichotomy: "*There is no dividing line between these two fields; there is only one mathematics*."

His dissertation, which he had discussed with RICHARD VON MISES, was titled *"The Difference Method with Higher Approximation for Linear Differential Equations"* and was in the field of *Numerical Analysis*. It was accepted by ALFRED KLOSE (actually an astrophysicist who did not understand much about the subject matter) and by ERHARD SCHMIDT. In the absence of any other offers, COLLATZ then accepted a position as a research assistant at the *Institute for Technical Mechanics* at the *Karlsruhe Technical University* (now *the Karlsruhe Institute of Technology* – KIT). Although he had no prior experience with this subfield, he quickly and successfully familiarised himself with the subject matter. After completing his habilitation in 1937 (*Proof of Convergence and Error Estimation for the Finite Difference Method for Eigenvalue Problems of Second and Fourth Order Ordinary Differential Equations*), he worked as a private lecturer in Karlsruhe.

With the outbreak of World War II, he was seconded to the *Institute for Practical Mathematics* at the Technical University of Darmstadt, where he carried out complex aerodynamic calculations for rocket development in Peenemünde. In 1943, he took up a position as full professor of mathematics at the Technical University of Hanover.

In 1952, he finally accepted a professorship in Applied Mathematics at the University of Hamburg, where he lectured until his death – even beyond his retirement in 1978. From 1963 to 1972, he was director of the University of Hamburg's computer centre. Years earlier, he had acquired an IBM 650 (the first mass-produced mainframe computer) for his institute, recognising early on the possibilities it offered for solving numerical problems, such as solving systems of equations with tens of thousands of variables.

COLLATZ was a prolific author, writing 36 books, many of which were translated into other languages. These included: *Eigenvalue Problems and Their Numerical Treatment* (1945), *Numerical Treatment of Differential Equations* (1951, 1955), *Functional Analysis and Numerical Mathematics* (1964), and *Differential Equations – An Introduction with Special Consideration of Applications* (1966). He was editor and co-editor of twelve scientific journals and published 227 high-quality scientific articles. He organised numerous conferences, including over 60 at the Oberwolfach Research Institute. During his long academic career, he received many honours, including honorary doctorates from the universities of São Paulo, Vienna, Dundee, London, Hanover, Augsburg, and Dresden. He was also a member of the Academies of Sciences of Bologna and Modena, the *Leopoldina*, the *Hamburg Mathematical Society* and the *National Geographic Society*.

When the German Mathematical Society (DMV) decided to publish a commemorative volume entitled "*A Century of Mathematics* 1890-1990" on the occasion of its 100th anniversary, only one author was suitable for the chapter on numerical analysis for the editors in charge: LOTHAR COLLATZ.

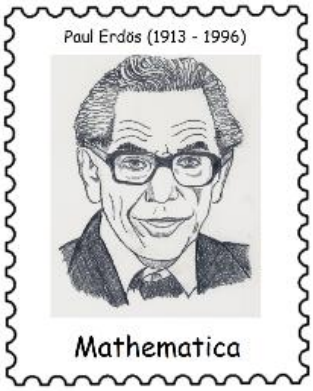
COLLATZ WAS well-respected and popular among his students for his friendly and approachable manner, not only among his 52 doctoral students. He strengthened his personal relationships through hikes on weekends spent together.

One aspect of his Curriculum Vitae that seems puzzling is his early enrolment, only three days after his state examination under VON MISES, in the SA (the *Sturmabteilung*, also known as the Brownshirts). This, along with his membership in the *Nazi party* (1937) and the *National Socialist German Lecturers' League*, can only be understood in the context of securing an academic career. In job applications, he concealed his close collaboration with RICHARD VON MISES, instead claiming that he had been significantly influenced by LUDWIG BIEBERBACH, the fanatical proponent of so-called *German Mathematics*, which was certainly *not* the case. During the denazification proceedings, COLLATZ described himself as *a scientist without political interests*. Initially, he was classified as *a follower* and ordered to pay a fine. However, upon seeing who else had been exonerated, he filed an appeal and subsequently, he too was exonerated. The Nazi files contain assessments of him such as "*too bourgeois"* and "*unmanly*" in the sense of Nazi ideology, but also, on the other hand, "*undoubtedly exceptionally gifted scientifically*" and "*possesses an unmistakable pedagogical talent*." After the denazification proceedings were completed, COLLATZ made no further comment on his behaviour during the Nazi era, instead always emphasising how much he owed to RICHARD VON MISES.

Although no famous theorems or methods are named after COLLATZ, a conjecture in number theory – which has not yet been resolved – is named after him. While searching for a simple example of a function that leads to a graph with a nontrivial cycle, he discovered the so-called $3n+1$-problem in 1937:

Start with any natural number *n*.
If *n is* even, halve the number; if *n is* odd, multiply it by 3 and add 1.

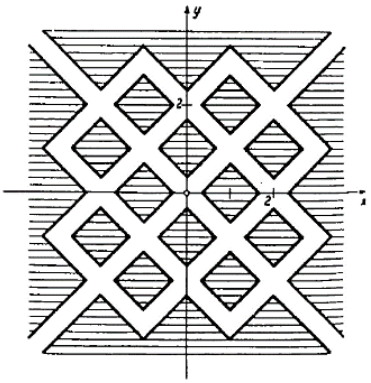
The **Collatz conjecture** states: This iterated sequence always results in cycle 4, 2, 1.



PAUL ERDŐS commented on the problem: *Mathematics is not yet ready for such problems. Hopeless. Absolutely hopeless.*

COLLATZ loved geometric patterns that result from inequalities – example (see figure on the right):

$$\big|\,|\,|x|-1\,|-1\,\big|-\big|\,|\,|y|-1\,|-1\,\big| \geq \tfrac{1}{3}\,,$$



(from: *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht, 1934)*

He invented numerous tessellations of the plane. In 1975, he noted that he had found 700 topologically distinct "plane divisions" to date.

COLLATZ was an extremely creative person, a gifted draftsman and painter; he invented dice and word games. From 1963 onward, he edited THOMAS RANDOW's contributions to the puzzle column *"Logelei"* in the weekly newspaper DIE ZEIT.

With his wife MARTHA, to whom he had been happily married since 1940, he travelled extensively and accepted numerous invitations to give lectures from around the world. He kept diaries of his experiences, illustrating them with sketches. At the end of each year, he compiled these into a greeting card, which he had reproduced and sent to his many friends. He had made several travel plans for 1990, which were tragically cut short by his sudden and unexpected death during a mathematics conference in Varna, Bulgaria.

In memory of LOTHAR COLLATZ, the ICIAM (*International Council for Industrial and Applied Mathematics*) awards the COLLATZ Prize every four years to a scientist under 42 years of age in recognition of outstanding work in the field of industrial mathematics and applied mathematics.

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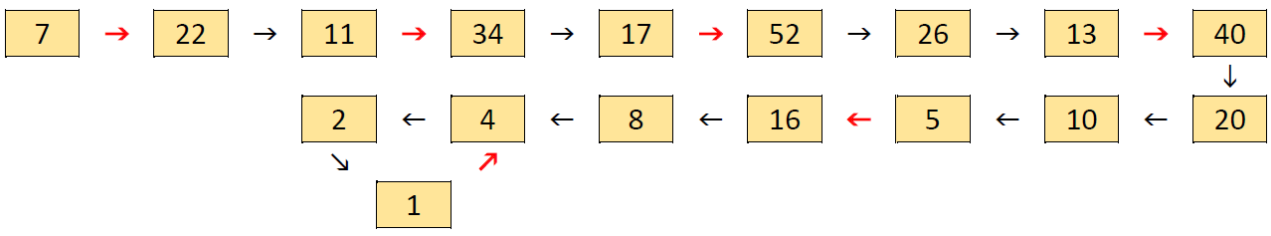
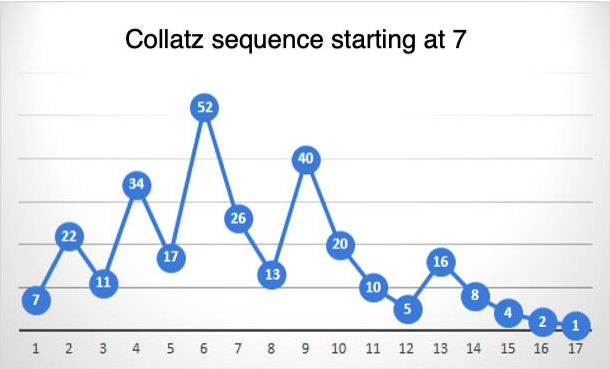
https://mathematik-ist-schoen.jimdoweb.com/

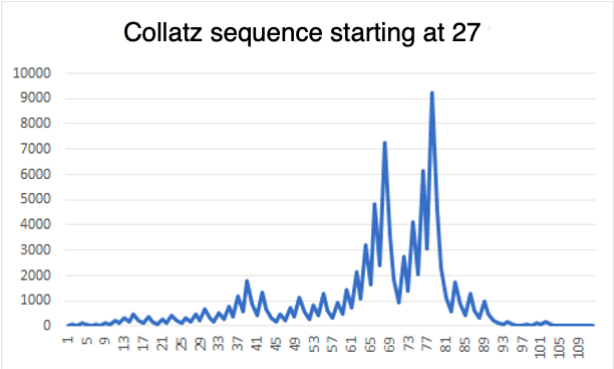
Translated by John O'Connor, University of St Andrews

Discoveries relating to Collatz's 3n + 1 problem

Example: Diagram for the starting number 7 with the 4-2-1 circle at the end:







The following excerpt from an Excel spreadsheet shows Collatz sequences with starting numbers between 1 and 30:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
4	1	10	2	16	3	22	4	28	5	34	6	40	7	46	8	52	9	58	10	64	11	70	12	76	13	82	14	88	15
2	4	5	1	8	10	11	2	14	16	17	3	20	22	23	4	26	28	29	5	32	34	35	6	38	40	41	7	44	46
1	2	16	4	4	5	34	1	7	8	52	10	10	11	70	2	13	14	88	16	16	17	106	3	19	20	124	22	22	23
	1	8	2	2	16	17		22	4	26	5	5	34	35	1	40	7	44	8	8	52	53	10	58	10	62	11	11	70
		4	1	1	8	52		11	2	13	16	16	17	106		20	22	22	4	4	26	160	5	29	5	31	34	34	35
		2			4	26		34	1	40	8	8	52	53		10	11	11	2	2	13	80	16	88	16	94	17	17	106
		1			2	13		17		20	4	4	26	160		5	34	34	1	1	40	40	8	44	8	47	52	52	53
					1	40		52		10	2	2	13	80		16	17	17			20	20	4	22	4	142	26	26	160
						20		26		5	1	1	40	40		8	52	52			10	10	2	11	2	71	13	13	80
						10		13		16			20	20		4	26	26			5	5	1	34	1	214	40	40	40
						5		40		8			10	10		2	13	13			16	16		17		107	20	20	20
						16		20		4			5	5		1	40	40			8	8		52		322	10	10	10
						8		10		2			16	16			20	20			4	4		26		161	5	5	5
						4		5		1			8	8			10	10			2	2		13		484	16	16	16
						2		16					4	4			5	5			1	1		40		242	8	8	8
						1		8					2	2			16	16						20		121	4	4	4
								4					1	1			8	8						10		364	2	2	2
								2									4	4						5		182	1	1	1
								1									2	2						16		91			
																	1	1						8		274			
																								4		137			
																								2		412			
																								1		206			

The sequence with starting number 27 stands out from the others: it consists of 112 elements, the largest element of this sequence is 9232 (incidentally, the sequences with starting numbers 31, 41, 47 also have this number as their largest value).

It is noticeable that several episodes with adjacent starting numbers are of the same length, e.g. 4 and 5, 12 and 13, 20 and 21, 28, 29 and 30 (and further 34 and 35, 36, 37 and 38, 44, 45 and 46, 49 and 50).

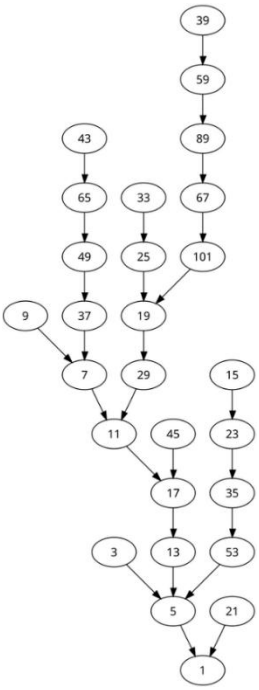
If the Collatz sequence with starting number n has length k, then the sequence with starting number $2n$ has length $k + 1$. If the Collatz sequence with odd starting number n has length k, then the sequence with starting number $3n + 1$ has length $k - 1$.

Odd terms of type $4n + 1$ (i.e., 5, 9, 13, 17, …) become even numbers of type $12n + 4$, then of type $6n + 2$ and then of type $3n + 1$ (even or odd).

Odd terms of type $4n + 3$ (i.e., 7, 11, 15, 19, …) become even terms of type $12n + 10$, these become odd terms of type $6n + 5$, these become even terms of type $18n + 16$ and then type $9n + 8$ (even or odd).

Odd terms of type 8*n* + 1 (i.e., 9, 17, 25, 33, …) become even terms of type 24*n* + 4, these then become even terms of type 12*n* + 2, then odd terms of type 6*n* + 1, these become even terms of type 18*n* + 4, then continue to type 9*n* + 2 (even or odd).

Odd terms of type 8*n* + 3 (i.e., 11, 19, 27, 35, …) become even terms of type 24*n* + 10, these then become odd terms of type 12*n* + 5, then even terms of type 36*n* + 16, these become even terms of type 18*n* + 8, then continue to type 9*n* + 4 (even or odd). …



The image *on the right* shows the directed graph for the starting numbers below 30 (except for 27), which includes only the *odd* numbers and omits the intervening even numbers of the Collatz sequence.

(Grahic: Wikimedia, Author: Keenan Pepper)

A more comprehensive directed graph by the same author is available at https://commons.wikimedia.org/wiki/File:Collatz-graph-300.svg

The image *below* shows a tree containing all number sequences that have a length of less than 20. (Graphic: Wikimedia, Author: Lovasoa)