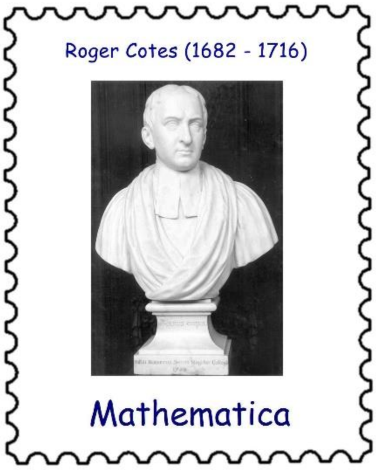
Roger Cotes (October 7, 1682 – June 5, 1716)

by Heinz Klaus Strick, Germany



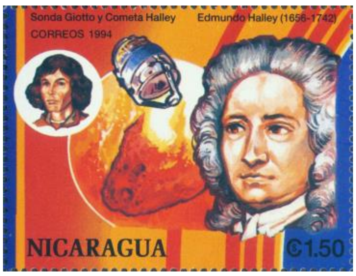
"If he had lived, we might have known something," Isaac Newton is said to have said when he learned of the death of Roger Cotes, who was only 33 years old. An unusual sentence from the outstanding genius who otherwise rarely had a word of recognition for his colleagues – surprising because the relationship between Newton and Cotes had recently cooled considerably.

Roger Cotes was born the second of three children of the headmaster of the parish of Burbage (located between Leicester and Coventry). While he was still at school in Leicester, his exceptional mathematical talent was noticed, which impressed his uncle, the Reverend John Smith, enough to personally look after his nephew's education. At the age of 12, he arranged for Roger to transfer to the famous St Paul's School in London. The lively correspondence between uncle and nephew in the following years mainly concerned mathematical subjects.

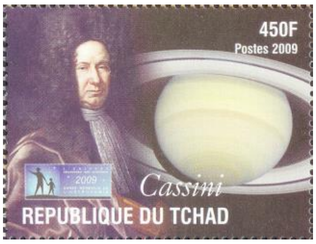
In 1699, Cotes was admitted as a paying student at Trinity College, Cambridge, where Isaac Newton only nominally held the position of *Lucasian Professor*. In fact, Newton worked as *Warden* from 1696 and as *Master of the Royal Mint* in London from 1700. He did not give up his position in Cambridge until 1701.

After his bachelor's degree in 1702, the outstanding Cotes received a fellowship to continue his studies. When in 1704 – thanks to a donation from Thomas Plume, Archdeacon of Rochester, the chair of *Plumian Professor of Astronomy and Experimental Philosophy* was established at the University of Cambridge, Richard Bentley, Rector of Trinity College, had no difficulty in convincing both Isaac Newton and William Whiston (his successor to the *Lucasian Chair*) that only one person was suitable for the position: Roger Cotes.

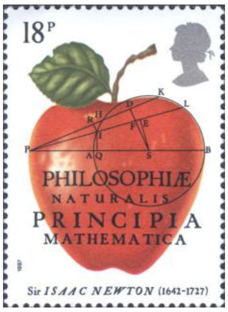
Only John Flamsteed, the *Astronomer Royal* of the Observatory in Greenwich and in constant dispute with Newton and Edmond Halley, opposed the appointment, as he considered his own assistant more suitable. In 1706, at the same time as he was awarded his MA degree, Cotes was appointed the first professor in the chair, which is still highly regarded worldwide today. (Among Cotes's successors are celebrities such as George Bidell Airy and Arthur Eddington.)



Over time, COTES set up an observatory above the *King's Gate* of the university and he also lived there, together with his cousin ROBERT SMITH, who became his assistant. After COTES's death, SMITH became the second holder of the chair. As an astronomer, COTES developed a telescope for observing the sun (with a clockwork drive) and revised the planetary tables of FLAMSTEED and JEAN - DOMINIQUE CASSINI.

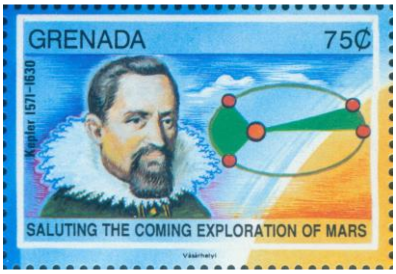


During a solar eclipse in April 1715, COTES was able to make important measurements of sunspots. However, in the short period of his activity, his contributions to astronomy fell short of the expectations placed on him when he was appointed. However, his (posthumously published) theoretical considerations on the optimal evaluation of series of measurements are worthy of mention – in principle, this is the method of minimising the sum of squares of errors.



In 1687, ISAAC NEWTON had only a few copies of his *Principia* (*Philosophiae Naturalis Principia Mathematica*) printed. In the meantime, several errors had been discovered in the work and, not only for this reason, a revision of the first edition seemed overdue.

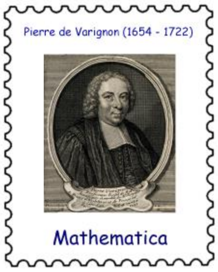
NEWTON, who had put all his energy into his new job as director of the Royal Mint, actually only wanted to make minor corrections in a new edition, but BENTLEY finally convinced him that a more extensive edit was necessary and that COTES could help him with that. Thus began an intensive collaboration between NEWTON and COTES in 1709 that lasted almost four years. COTES presented NEWTON with numerous changes and additions to the text, which NEWTON was initially hesitant about but then largely accepted. COTES also wrote an additional foreword to the new edition. In this he showed that the available observational data regarding the orbit of the moon confirmed NEWTON's law of gravitation and KEPLER's third law, and how comet orbits could be described by conic sections (with the sun at the focus). He also explained why RENÉ DESCARTES' vortex theory, which many people (including LEIBNIZ) still consider to be correct, could not be correct.



In 1713, the new edition of the *Principia* finally appeared in an edition of 750 copies. (In addition, other books were printed as pirated copies in Amsterdam.)

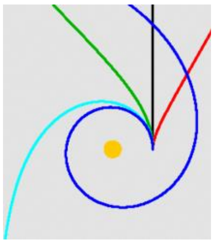
COTES received neither remuneration nor thanks for his intensive work as co-author. In the original version of NEWTON's foreword, NEWTON had formulated words of thanks, but deleted them again when a disagreement arose between the two at the end of the collaboration. NEWTON accused COTES of having overlooked a serious error in the first edition of the *Principia.*

It was JOHANN BERNOULLI, a mathematician from the LEIBNIZ camp, who discovered this error during the editing phase of the new edition. (Incidentally, after the completion of the 2nd edition, NEWTON sent several free copies to mathematicians on the continent, including PIERRE DE VARIGNON , but none to JOHANN BERNOULLI – on the grounds that he had too few copies.)



During his short life, COTES, a member of the *Royal Society* since 1711, was able to publish only one paper: *Logometria,* published in 1714 in the *Philosophical Transactions* . It contains, among other things, the continued fraction expansion $[2;1,2,1,1,4,1,1,6,1,1,8,1,1,...]$ for the base *e* of NAPIER's logarithms and, 34 years before LEONHARD EULER, the derivation of the equation $i\cdot\varphi=\log\left(\cos(\varphi)+i\cdot\sin(\varphi)\right)$

COTES died – probably after contracting typhus – at the age of less than 34. He left behind a wealth of manuscripts that show the enormous progress his subsequent work could have made in mathematics. His cousin ROBERT SMITH tried to organise the papers and he published some of them in 1722 under the title *Harmonia mensurarum*. This document contains, among other things, extensive tables with antiderivatives of various rational functions, which demonstrate how confidently COTES could handle logarithmic, trigonometric and hyperbolic functions. He used the antiderivatives to determine arc lengths of special curves as well as surfaces and volumes of solids of revolution.



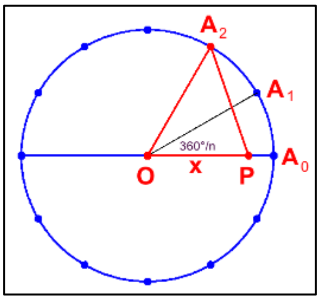
He also investigated possible trajectories on which bodies can move when subjected to centripetal forces that are inversely proportional to the cubes of their distances (so-called COTES spirals, see Wikipedia figure on the right by WillowW).

COTES discovered, among other things, the following relationships between geometry (trigonometry) and algebra; these were further developed by ABRAHAM DE MOIVRE in 1730.

$$x^{2n}+1=\left[x^2-2x\cdot\cos(\tfrac{1}{2n}\cdot\pi)+1\right]\cdot\left[x^2-2x\cdot\cos(\tfrac{3}{2n}\cdot\pi)+1\right]\cdot\ldots\cdot\left[x^2-2x\cdot\cos(\tfrac{2n-1}{2n}\cdot\pi)+1\right].$$

Examples: $x^4+1=(x^2-\sqrt{2}\cdot x+1)\cdot(x^2+\sqrt{2}\cdot x+1),\ x^6+1=(x^2-\sqrt{3}\cdot x+1)\cdot(x^2+1)\cdot(x^2+\sqrt{3}\cdot x+1)$.

COTES' theorem : For *n* equally spaced points on the unit circle $A_0, A_1, A_2, ...A_{n-1}$ and a point *P* lying on OA_0 with $OP=x$ the following applies: $PA_0\cdot PA_1\cdot PA_2\cdot\ldots\cdot PA_{n-1}=1-x^n$.



Proof idea: Since the points $A_0, A_1, A_2, ...A_{n-1}$ are symmetrical about the *x* -axis, quadratic factors appear in the product on the left, for which the following applies according to the cosine theorem: $PA_k^{\ 2}=1-2x\cdot\cos\left(\frac{2k\pi}{n}\right)+x^2$. For example, for $n=3$ we get:

$$PA_0\cdot PA_1\cdot PA_2=PA_0\cdot PA_1^{\ 2}=(1-x)\cdot(1-2x\cdot\cos\left(\tfrac{2\pi}{3}\right)+x^2)=(1-x)\cdot(1+x+x^2)=1-x^3$$

and for $n=4$:

$$PA_0\cdot PA_1\cdot PA_2\cdot PA_3=PA_0\cdot PA_1^{\ 2}\cdot PA_2=(1-x)\cdot(1-2x\cdot\cos\left(\tfrac{2\pi}{4}\right)+x^2)\cdot(1+x)=(1-x^2)\cdot(1+x^2)=1-x^4 .$$

Cotes also provided formulas for the approximate calculation of surface areas by subdividing the areas with equidistant support points (leading to the so-called NEWTON - COTES formulas), which were published in 1750 by THOMAS SIMPSON in *The Doctrine and Applications of Fluxions.*

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