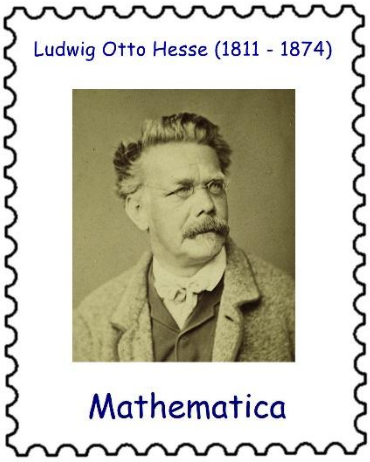
OTTO HESSE (April 22, 1811 - August 4, 1874)

by HEINZ KLAUS STRICK, Germany



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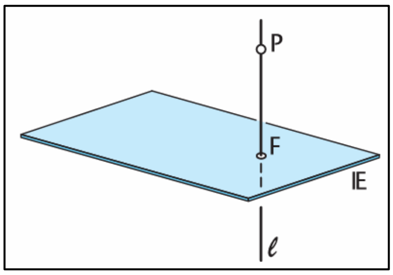
One of the compulsory topics in the upper secondary school curriculum, within the framework of analytical geometry, is the treatment of coordinate equations describing planes in 3-dimensional space. From the coordinate equation

$ax_1 + bx_2 + cx_3 = d$ of a plane with a normal vector $\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$,

one obtains $\dfrac{ax_1 + bx_2 + cx_3 - d}{\sqrt{a^2 + b^2 + c^2}} = 0$ *the Hesse normal form of the plane*, named after the mathematician LUDWIG OTTO HESSE.

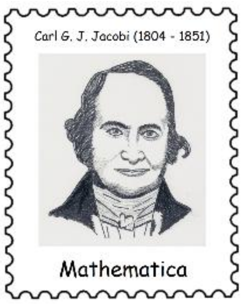
This form of a plane equation is useful when the distance of a point $P(p_1, p_2, p_3)$ from the plane *E* is to be determined (= length of the perpendicular section *PF*):

$$\text{Distance}(P, E) = \left| \frac{ap_1 + bp_2 + cp_3 - d}{\sqrt{a^2 + b^2 + c^2}} \right|$$



LUDWIG OTTO HESSE was born in 1811, the eldest of five children of a wealthy merchant and brewer, in the East Prussian university town of Königsberg (now Kaliningrad in an exclave of Russia). While still attending the *Altstädtisches Gymnasium* (Old Town Gymnasium), he demonstrated a particular talent for mathematics and the natural sciences, which was further encouraged by the employment of an astronomy student as his private tutor. His happy childhood and youth were abruptly cut short in 1829 when his father died after falling from a horse. After passing his school leaving examination in April 1832, OTTO was exempted from his one-year military service because the regimental doctor, Dr CLEBSCH, deemed him too flat-chested, allowing him to immediately enrol to study mathematics and the natural sciences at the University of Königsberg. (Incidentally, 22 years later, CLEBSCH's son ALFRED studied under OTTO HESSE.)

HESSE was fortunate that one of his mathematics professors was CARL GUSTAV JACOB JACOBI, arguably one of the most important mathematicians in Germany at the time. He fascinated his students with the structure of his lectures, which he delivered with great clarity and went beyond the mere content of the textbooks.



HESSE was also influenced by the astronomy lectures of FRIEDRICH WILHELM BESSEL, who had taught in Königsberg since 1810 and who knew how to combine practice with theory.

In May 1837, Hesse passed the senior teacher examination, which authorized him to teach mathematics and physics in the upper grades of the Gymnasium. After a probationary year at the Stadtgymnasium (comparable to a traineeship), the young teacher was certified *for his pedagogical zeal, practical teaching ability, and scientific spirit* – with the prognosis that *he justified very favourable expectations in every respect*.

After this phase of training, Hesse went on a journey for five months, via Berlin, Dresden, Salzburg, Innsbruck to Venice and back again via Strasbourg and Heidelberg. The city on the Neckar fascinated him so much that he dreamed for years of being able to return there one day.

From October 1837 onwards, he taught eight hours a week at a trade school, while simultaneously continuing the research into second-order curves and surfaces he had begun during his studies.

After completing his thesis, he received his doctorate in January 1840. His dissertation, *"De octo punctis intersectionis trium superficium secundi ordinis"* (On the eight points of intersection of three second-order surfaces), served as the basis for his habilitation in 1841. He prefaced his work as a private lecturer with the motto "*Praecipuum docentis officium est docere discendi vias"* (The most important task of the teacher is to demonstrate the paths of learning).

In 1841, he married Marie Sophie Emilie Dulk, daughter of a chemistry professor at the University of Königsberg. The happy marriage produced six children; the firstborn son, who died in his first year, was followed by five daughters.

From 1843 onwards, Hesse took over the lectures of Jacobi, who had given up his teaching post in Königsberg for health reasons. The income from giving lectures was insufficient to support his young family, so Hesse was forced to take on additional teaching duties at a school.

This initially remained unchanged when he was appointed *Extraordinary Professor in October 1845.* Although this entailed a teaching obligation and the associated examination duties, it was initially unpaid. Nevertheless, Hesse fulfilled his obligations conscientiously; among the students he supervised were Gustav Robert Kirchhoff, Alfred Clebsch and Rudolph Lipschitz.

After unsuccessful applications for a professorship in Dorpat (now Tartu in Estonia), which failed due to the veto of the Russian authorities, and at the newly founded Polytechnic in Zurich (now ETH), the call to Halle finally came in 1855.

Hesse only stayed there for one semester, however, because during the course of that semester, he received two appointments: from the *Gewerbeinstitut* in Charlottenburg (now the *Technische Hochschule Berlin*) and from the University of Heidelberg. He accepted the call to Heidelberg without hesitation.

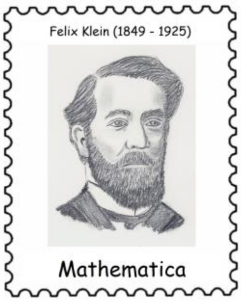
There he enjoyed the collegial collaboration of Hermann Helmholtz, Robert Bunsen, and his former student Kirchhoff. His lectures were always well attended, as he knew how to inspire his students with the *divine science* of mathematics.

He also received recognition from various institutions, including membership in the Göttingen and Berlin Academy of Sciences. When he received a simultaneous appointment to the University of Bonn and the newly established *Polytechnic* in Munich (now the *Technical University*) in the summer of 1868, he assumed this would improve his financial situation in Heidelberg, but this did not happen. He therefore moved to Munich for the final years of his life, where he was primarily involved in the reform of teacher training.

During his time in Munich, his health deteriorated rapidly due to a liver disease; even a spa stay in Carlsbad brought no relief. He died on August 4, 1874; the funeral took place in the Heidelberg cemetery with a large number of former colleagues and students in attendance.

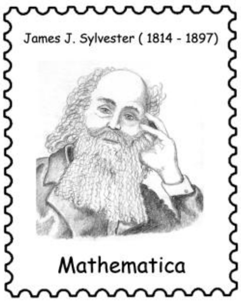
As a mathematician, OTTO HESSE worked primarily in the fields of algebra, analysis, and analytic geometry. The *Bavarian Academy of Sciences*, to which HESSE was admitted in 1868, posthumously published his complete works, including numerous contributions that had first appeared in *Crelle's Journal für die reine und angewandte Mathematik*.

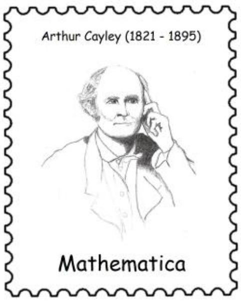
His textbooks, *Lectures on Analytic Geometry of Space* and *Lectures on Analytic Geometry of the Straight Line, the Point, and the Circle in the Plane,* were widely used; the aforementioned *Hesse normal form* (and, analogously, that of a straight line) were included in these books.



As FELIX KLEIN later wrote, HESSE spent his most creative years in Königsberg, where he focused primarily on the geometric properties of algebraic curves and surfaces. The scientific community first became aware of the young scientist in 1844 through two publications in *Crelle's Journal*: *On the Elimination of Variables from Three Second-Degree Algebraic Equations with Two Variables* and *On Inflection Points of Third-Order Curves*.

Incidentally, HESSE himself considered his 1856 contribution *On the Double Tangents of Fourth-Order Curves* to be his most successful publication (double tangents touch the graph at two points).

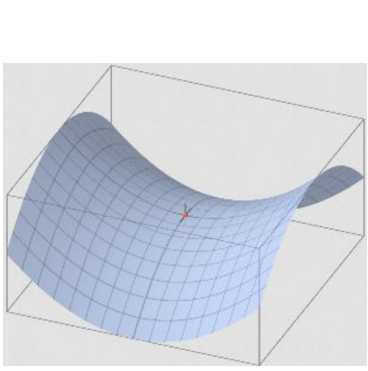




Today, several technical terms commemorate OTTO HESSE and are also used internationally, such as the so-called *Hesse matrix* (*Hessian matrix*, *Matrice hessien*, *Matrice hessiana*).

The term originates from a suggestion by JAMES JOSEPH SYLVESTER, who, together with ARTHUR CAYLEY, was working on similar questions at the same time.

This matrix can be used to describe the local curvature behavior of a twice differentiable function $f\colon D \subset \mathbb{R}^n \to \mathbb{R}$. In the 2-dimensional case, it is a matter of representing the partial derivatives in the form of the matrix $H_f = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2 \partial x_2} \end{pmatrix}$.



According to SCHWARZ's theorem, such a matrix is symmetric. From the corresponding *Hessian determinant* $\Delta H_f = \frac{\partial^2 f}{\partial x_1 \partial x_1} \cdot \frac{\partial^2 f}{\partial x_2 \partial x_2} - \left(\frac{\partial^2 f}{\partial x_1 \partial x_2}\right)^2$ one can read off, among other things, whether a point of the graph is an extreme point or a saddle point, as for example in the graph of $f(x,y) = x^2 - y^2$ (see the Wikipedia figure on the right).

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https://www.spektrum.de/wissen/das-leben-des-mathematikers-otto-hesse/2246357

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