## **CAMILLE JORDAN** (January 5, 1838 – January 22, 1922)

by HEINZ KLAUS STRICK, Germany

MARIE ENNEMOND CAMILLE JORDAN came from a wealthy and respected family. His father, a graduate of the *École Polytechnique*, worked as an engineer and was a deputy for the department of Saône-et-Loire, and his mother's brother was one of the most important muralists in France in the second half of the 19th century.

At the age of 17, CAMILLE passed the entrance examination to the *École Polytechnique* at the top of his class. As was customary in those days, his studies included not only mathematical theories but also training as an engineer, so that after completing his studies,



CAMILLE JORDAN – like AUGUSTIN CAUCHY and HENRI POINCARÉ – worked as an engineer, first in the provinces and later in Paris.



In 1861 JORDAN received his doctorate; his work consisted of two parts: *Sur le nombre des valeurs des fonctions* (On the number of function values) from the algebra section and *Sur des periodes des fonctions inverses des intégrales des différentielles algébriques* (On the periods of inverse functions of the integrals of algebraic differentials) from the analysis section.

In 1862, JORDAN married MARIE-ISABELLE MUNET, the daughter of the Deputy Mayor of Lyon. The happy marriage produced two daughters and six sons. In addition to his work as an engineer, JORDAN had plenty of time to study mathematical questions from almost all areas of research.

In his first publications, he studied the symmetry properties of polyhedra, among other things.

In 1870 he published *Traité des substitutions et des équations algébraique* (Treatise on substitutions and algebraic equations), a 667-page work that gave a comprehensive account of GALOIS theory and the theory of groups.



This contained, among other things, a theorem on normal series of groups (ascending chains of subgroups with special properties), which was later named after JORDAN. This theorem was extended by OTTO LUDWIG HÖLDER in 1889 and is therefore cited in the literature as the JORDAN-HÖLDER theorem.



For this treatise JORDAN received the Prix PONCELET of the *Académie des Sciences* (donated by PONCELET's widow).

In 1846 JOSEPH LIOUVILLE was the first to recognise the importance of GALOIS' work and to lecture on it, with the participation of JOSEPH BERTRAND and CHARLES HERMITE, who carried out further research.

After JORDAN's publication, a number of mathematicians came to Paris to talk directly to JORDAN and to get inspiration for their own research, including SOPHUS LIE and FELIX KLEIN. JORDAN continued his research on groups in the decades to come, including matrix groups and their subgroups.





Named after JORDAN is the so-called *JORDAN normal form of matrices*, see the Wikipedia figure on the right.

From 1873 JORDAN was an examiner at the *École Polytechnique*. In 1876 he was appointed to the chair of analysis. In 1881 he was elected a member of the *Académie des Sciences*.

From his lectures on analysis he developed a three-volume work, the *Cours d'Analyse de l'École Polytechnique*. The first edition was published in 1887; further editions followed in 1887 and between 1909 and 1915.



However, the level of difficulty of these books on analysis goes far beyond that of his lectures, which were aimed at engineering students for whom it was less important to have a general and precisely founded theory (as is the case in the three volumes) than a comprehensive presentation of the applications of differential and integral calculus.

When asked about this discrepancy by HENRI LEBESGUE, JORDAN apologised by pointing out that the sole purpose of the title was to satisfy the publisher. Lectures, in which theory was emphasised even more strongly, were then given by JORDAN at the *Collège de France* (as successor to LIOUVILLE) from 1883.

The third volume of his *Cours d'Analyse* contains a famous result which is named after him today. At first sight, the result seems obvious; JORDAN, however, showed the need for a proof. The proof he developed was later criticised by some mathematicians (including the American mathematician OSWALD VEBLEN, who then made a more precise statement himself).

• The JORDAN curve theorem:

Every closed non-intersecting JORDAN curve in the Euclidean plane divides it into two disjoint regions whose common edge is the JORDAN curve and whose union with the JORDAN curve is the whole plane. Exactly one of the two regions is bounded.

A graph is called a *closed JORDAN curve* if its points can be described by a continuous representation of parameters and if the start and end points are identical.

*Example*: The parameterisation of the circle by  $\varphi(t) = (\cos(t), \sin(t))$  with  $t \in [0, 2\pi]$  and  $\varphi(0) = \varphi(2\pi)$  meets this condition.



Fractal curves can also fulfil this condition (see the first three examples below). In the continuous loop on the right, on the other hand, it is hardly possible to tell which of the points are inside and which are outside.



Independently of GIUSEPPE PEANO, he developed criteria for determining a measure of any set of points in the plane and in higher-dimensional spaces. He studied the graphs of oscillating functions by introducing the criterion of *bounded variation*. In this context he proved the theorem that only graphs with bounded variation are rectifiable, i.e. have a finite arc length.

For example, the graphs of  $f_0(x) = \sin(\frac{1}{x})$  (*left*) and  $f_1(x) = x \cdot \sin(\frac{1}{x})$  (*centre*) on the interval  $]0, \frac{2}{\pi}]$  are of unbounded variation, whereas the graph of  $f_2(x) = x^2 \cdot \sin(\frac{1}{x})$  (*right*) is of bounded variation.



JORDAN received numerous honours: as early as 1869 he was elected as a corresponding member of the *Göttingen Academy of Sciences*. After his retirement (1912) he became President of the *Académie des Sciences* (1916), Foreign Member of the *Royal Society* (1919), Honorary President of the International Congress of Mathematicians in Strasbourg (1920). After the death of LIOUVILLE (1882), he became editor of the *Journal de Mathématiques Pures et Appliquées* (also known as the LIOUVILLE Journal). These honours could do little to console JORDAN for his personal losses: three of his sons and a grandson died as soldiers during the First World War. CAMILLE JORDAN died of heart failure in 1922.

A final anecdote: At the beginning of the 1900 Congress of Mathematicians in Paris, when those who had died since the last meeting were commemorated, the name of CAMILLE JORDAN was announced and the date of his death was given as 7 November 1898. JORDAN, who was present, is said to have stood up and remarked that at least the year could not be correct.

*Note*: The so-called *GAUSS-JORDAN method* for the solution of a linear system of equations is named after CARL FRIEDRICH GAUSS and the German geodesist WILHELM JORDAN, and so-called *JORDAN algebras* are named after the German quantum physicist PASCUAL JORDAN, neither of whom is related to CAMILLE JORDAN.

First published 2020 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/camille-jordan-verfrueht-fuer-tot-erklaert/1693182

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