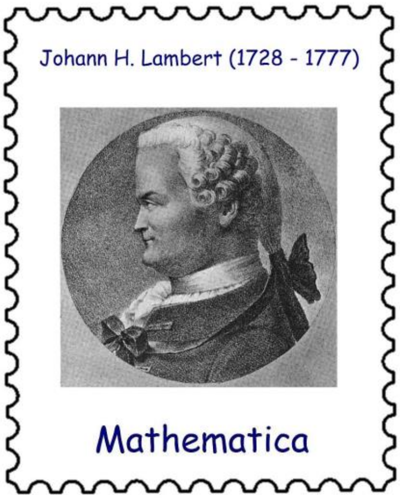
JOHANN HEINRICH LAMBERT (August 26, 1728 – September 25, 1777)

by HEINZ KLAUS STRICK, Germany



If you look at the conditions under which JOHANN HEINRICH LAMBERT spent the first years of his life, you can only marvel at what he became. His family, originally from Lorraine, had settled in the free imperial city of Mulhouse because they could profess their Calvinist faith there.

JOHANN HEINRICH was one of seven children of a tailor (five sons, two daughters). At least he had the opportunity to attend school until he was 12 years old, but then he had to work in his father's workshop so that the family could survive financially.

But instead of resting from the hard work in the evenings, he took every opportunity to study.

When he was 15, an opportunity to contribute more to the family's upkeep presented itself. Since he had nice handwriting, he could hire himself out as a clerk in the office of an ironworks and two years later he was employed as a secretary by the publisher of the *Basler Zeitung*. Here he now had the opportunity to access all the sources of subjects that interested him: Mathematics, astronomy, philosophy.

When he was 20 years old, he moved to Chur to work as a tutor for Count PETER VON SALIS. From then on he looked after the count's 11-year-old grandson, his cousin of the same age and another 7-year-old boy in the family.

LAMBERT now had a well-stocked library at his disposal, which he used for his studies. In his free time he built his first astronomical measuring instrument. In Chur he made contact with a discussion group where the latest scientific findings were discussed. Soon he was also accepted into the literary society of Chur and elected to the Swiss *Société Scientifique* based in Basel.

A first contribution appeared in 1755 in the *Acta Helvetica* on the *theory of heat*. The following year, LAMBERT set off on an academic grand tour of Europe with his two protégés, now 18 years old and his first destination was Göttingen.

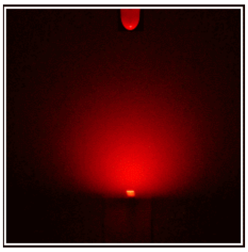
When French and Austrian troops occupied the university town of Göttingen at the beginning of the *Seven Years' War*, LAMBERT had to interrupt his studies there. However, he continued to cultivate his contacts with the Göttingen *Academy of Sciences*, to which he had been admitted. In order not to become involved in warlike activities, he travelled on with the two young people to Utrecht and from there to other Dutch cities.

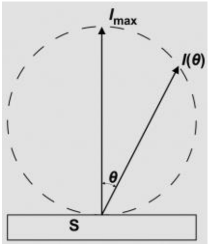
In 1758, LAMBERT's first book was published in The Hague, in which he presented the results of his experiments on the passage of light through various media.

LAMBERT completed his Grand Tour with visits to Paris, where he exchanged ideas with JEAN LE ROND D'ALEMBERT, and returned to Chur via Marseille, Nice, Turin and Milan.

His hopes of finding a job at Göttingen University were dashed. After making astronomical observations in Zurich, he travelled to Augsburg, where he published two books on photometry in 1760.

As early as 1729, PIERRE BOUGUER, one of the leaders of the famous longitude expedition to South America, had discovered that the strength of a beam of light decreased exponentially in an absorbing medium, and although LAMBERT paid appropriate tribute to BOUGUER's merits in his books, the name *LAMBERT's law of absorption* is the most common today. In another law, *LAMBERT's cosine law*, LAMBERT described the decrease in radiant intensity as the angle of radiation becomes flatter.





In the illustration on the left (from Wikipedia), a laser beam is directed from above onto a strip of paper; the strength of the scattered light is at a maximum when the scattering is perpendicular to the paper strip.

In the USA, a "Lambert" is still commonly used as the unit of measurement for luminance.

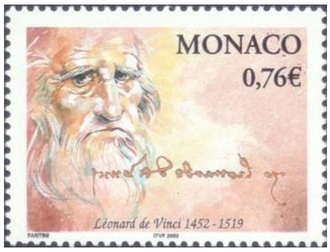
In 1760 LEONHARD EULER recommended him as Professor of Astronomy for a long-vacant post in St Petersburg, but the post was not filled as the *Academy* there was reorganised.

In 1762, LAMBERT was commissioned to establish a Bavarian *Academy of Sciences* in Munich (modelled on the Prussian *Academy* in Berlin) but after quarrels with other members of the project, he gave up this post. In the meantime, his *Cosmological Letters* were published, in which he developed his ideas about the structure of the universe. In Leipzig he put his philosophical treatise *Neues Organon* into print.

In 1764 a dream finally seemed to come true: LAMBERT accepted EULER's invitation to Berlin, hoping for a scientific collaboration with his great role model. Instead, a dispute soon developed between the two over questions of financing the *Academy* – possibly one of the reasons for EULER's decision to leave Berlin and return to St Petersburg. The Prussian ruler also initially found it difficult to assign LAMBERT an academic post. LAMBERT's strange clothes and his exaggeratedly submissive behaviour irritated his fellow men. But after FRIEDRICH II also recognised LAMBERT's scientific abilities, he was able to work as a member of the physical class (department) of the *Academy*. However, LAMBERT made his living primarily from his income as a senior civil engineer.

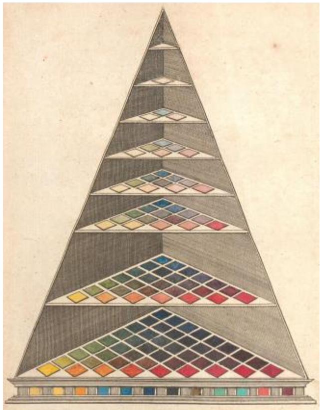
In the remaining twelve years of his life, he published over 150 papers, which were published by the *Academy*. After LAMBERT's death, JOHANN III BERNOULLI published four volumes of his extensive correspondence with other scholars.

There is hardly a scientific field in which the versatile and interested scholar did not become active. His writings on philosophy and logic impressed IMMANUEL KANT, who described him as a man of "resolute perspicacity and generality of insight". KANT intended to dedicate his *Critique of Pure Reason* to LAMBERT, but refrained from doing so when LAMBERT died before his work was published.



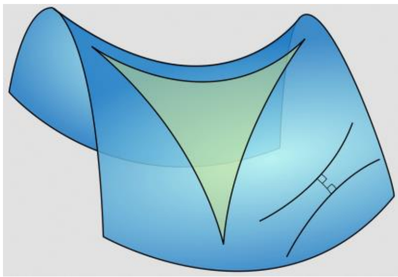
To carry out his metereological and astronomical observations, Lambert developed his own measuring instruments. In 1755, for example, he invented the first usable hygrometer to measure the humidity of the air (Leonardo Da Vinci's invention from 1480 had fallen into oblivion).

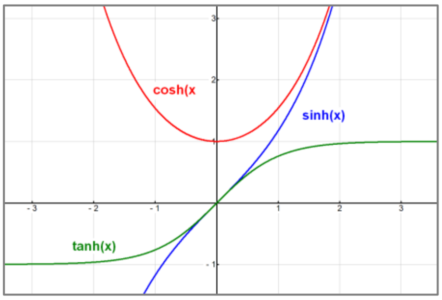
In 1759, he published a method for finding a perspective representation of an object or a building without first drawing a ground plan of the object - several years before Gaspard Monge's publications on descriptive geometry - and he invented a *perspectograph* with which such a drawing was possible.



Inspired by the colour triangle of the Göttingen mathematician and astronomer Tobias Mayer, who died in 1762, Lambert presented his three-dimensional colour pyramid in 1772, showing how mixing the three primary colours red (*vermilion*), yellow (*golden yellow*) and blue (*azurite*) produced $45+28+15+10+6+3=107$ different colours. The colours became lighter towards the top; at the uppermost layer, this resulted in the colour white. At the bottom, frequently used colours were shown in a bar. The aim of his construction was to facilitate communication between merchants, craftsmen (dyers, painters, printers) and their customers through this colour swatch; but natural scientists could also benefit from his scale.

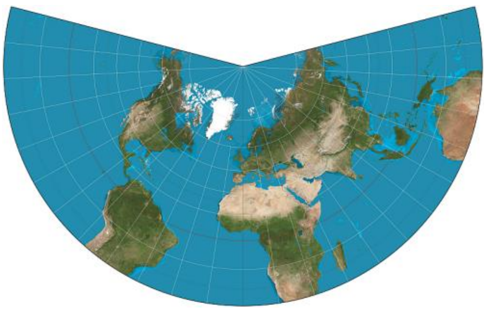
Lambert dealt intensively with the problem of producing undistorted maps. In this context, he investigated the validity of geometric theorems on curved surfaces, and he recognised that the larger the area Δ, the more the sum of angles in a *hyperbolic triangle* was less than 180°, and vice versa.
The equation $\pi-(\alpha+\beta+\gamma)=C\cdot\Delta$ with a proportionality factor *C* is called *Lambert's area formula*.



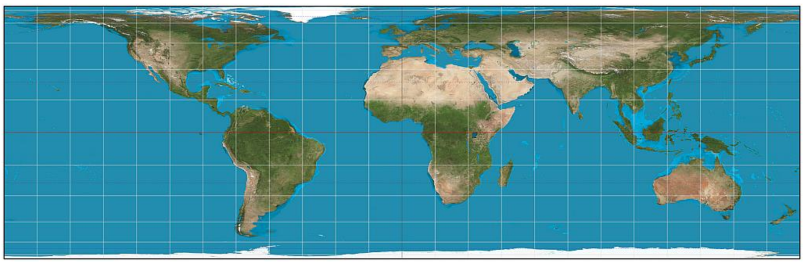


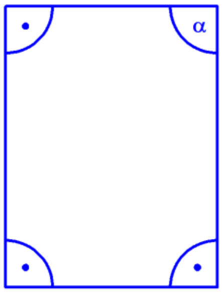
For his calculations he defined (independently of Vincenzo Riccati) the hyperbolic functions sinh, cosh and tanh.

(graphics from Wikipedia)



Lambert explained why it was not possible to draw maps on which all lengths, angles and area sizes are preserved – distortions compared to reality are therefore unavoidable. In 1762 he published seven projection methods, among others the *conical projection* (preserving angle), the *azimuthal projection* (preserving area) and the *cylindrical projection* (also preserving area), which are still used today.





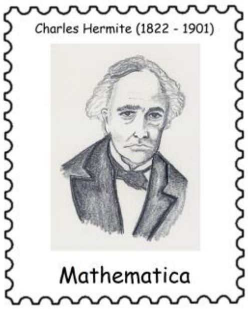
The theory of parallel lines, written in 1766, but not published posthumously until 1786, is remarkable. In this he dealt with EUCLID's parallelism axiom. Even before LAMBERT, mathematicians had dealt with the question of whether EUCLID's 5th axiom could be replaced by another one. LAMBERT continued the idea of the Italian mathematician GIROLAMO SACCHERI from 1733.

He investigated what could be inferred if the angle α occurring in (what is now called) *LAMBERT's quadrilateral* was obtuse, right-angled or acute. If α were obtuse, then this would contradict the other four EUCLIDean axioms, however, this would be possible on the sphere. If α = 90°, then this is equivalent to the axiom of parallelism. In his attempt to lead the third case to a contradiction, he got further than any of his predecessors, but he shied away from the decisive step that could have led to a non-EUCLIDean geometry. How close he was to this became clear from his statement that an acute angle α would only be possible on the surface of a sphere with an imaginary radius.

Although LAMBERT considered himself not particularly competent in the methods of analysis, he was the first to succeed in proving the irrationality of the circle number π. Starting from the series development of the sine and cosine functions, he obtained the representation of the tangent series in the form of an infinite continued fraction by applying the EUCLIDean algorithm for the series division:

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \ldots}{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \ldots} = \cfrac{x}{1 - \cfrac{x^2}{3 - \cfrac{x^2}{5 - \cfrac{x^2}{\ldots}}}}$$

Hence he could show that in the case of a rational number on the right side of the equation an irrational number resulted. And since $\tan(\frac{\pi}{4}) = 1$ is a rational number, the irrationality of $\frac{\pi}{4}$ and thus of π follows. LAMBERT's proof was very elaborate and could only be slightly simplified even by CARL FRIEDRICH GAUSS. Eventually CHARLES HERMITE found a shorter method of proof in 1873.



(drawing © Andreas Strick)

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