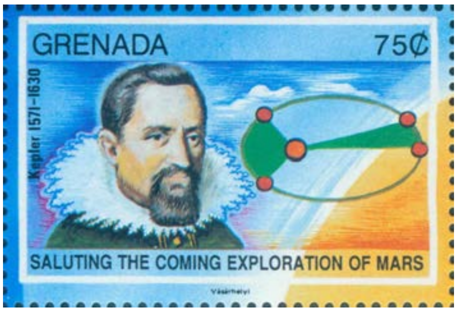
JOHN NAPIER (1550 – April 4, 1617)

by HEINZ KLAUS STRICK, Germany

The "invention" of logarithms by JOHN NAPIER, LAIRD OF MERCHISTON, at the beginning of the 17th century had a major impact on the development of the natural sciences.

The definition equation $e^{\ln N} = N$ (this formulation is not credited to NAPIER, but to LEONHARD EULER) is one of the *ten mathematical formulas that changed the face of the earth* – at least according to the assessment of the Nicaraguan postal administration.

There is no doubt that the introduction of logarithms has spurred the development of science. JOHANNES KEPLER recognized the tremendous advantage of his extensive calculations for determining the Martian orbit and ensured that the method was spread quickly with his own tables. Two hundred years later, LAPLACE wrote that this discovery "*by shortening the labours, doubled the life of the astronomer*".



(drawing © A. Strick)

Around 1600 the time was ripe for such a discovery. Mathematicians from the Islamic cultural area had already recognized the connections in principle, and the German mathematician MICHAEL STIFEL (1487 – 1567) used the formulation:

> *Addition in the arithmetic series corresponds to multiplication in the geometric, as well as subtraction in that of division in this. The simple multiplication in the arithmetic series becomes a multiplication in itself* (*exponentiation*) *in the geometric series. The division in the arithmetic series is assigned to the extraction of the roots in the geometric series, like the halving to the extraction of square roots.*

However, this restricted the application of the power laws to integer exponents, while JOHN NAPIER extended this to any exponent.

The ingenious Swiss watchmaker and instrument maker JOST BÜRGI (1552 – 1632), who worked as court astronomer to the Landgrave of Hesse in Kassel from 1579, also dealt with the question of how to reduce the time required for the extensive calculations required in astronomy .

JOST BÜRGI's discovery of logarithms in 1588 and the first logarithm tables he created received little attention because he did not speak the scientific language of that time, Latin, and thus had no access to the forums of science.

It was not until 1620, after Napier's writings had already found widespread use, that he published his *Arithmetic and Geometrical Progress Tabules, as well as thorough instruction, on how these should be useful in all sorts of calculations*. The edition was destroyed during the Thirty Years' War.

John Napier grew up in a wealthy and influential Scottish aristocratic family. His father, Archibald Napier, was married by his parents at the age of 15 and his first son, John, was born when Archibald was 16 years old. (Incidentally, the spelling of the family name, as it is common today, is not found in the documents of the 16th and 17th centuries; rather, the spelling Neper is used very often, and also Napeir, Naipper and others.)

(Photo: Wikimedia)

At age 13, John was sent to *St Andrews University*. The rector looked after the boy personally, especially after his mother died. It is not known when John left to continue his studies on the continent (France, the Netherlands and possibly also Italy). He did not return to Scotland until 1571, when he got married, had two children with his first wife and ten children with the second.

He worked intensively on the cultivation of his lands and developed methods of improving the yield of the fields through fertilisation with minerals.

He also delved deeply into Theological Questions. He fanatically took the side of Protestantism and in 1593 wrote the *Plaine Discovery of the Whole Revelation of St John*, in which he tried to prove that the Pope in Rome was really the Antichrist. He also “calculated” the time of the Judgment Day (“between 1688 and 1700”). The book was widely distributed, including translations for the Netherlands, France and Germany (a total of 21 editions) and Napier considered it his most important achievement.

In order to prevent the threatening invasion by the Spanish Armada in Scotland, he designed new weapon systems, such as armoured vehicles, and investigated how the idea of the Archimedes to set sailing ships on fire with the help of burning mirrors could be realised. Even after the fall of the Armada (1588) there was a risk of invasion, as Scottish nobles, including his father-in-law, continued to seek an alliance with Spain directed against England.

It is not clear when and how Napier's particular interest in mathematics was aroused and he pursued this “hobby” with great intensity. In particular, he was interested in developing methods that could reduce computational effort. He worked on the problem for twenty years, and then in 1614 his work *Mirifici Logarithmorum Canonis Descriptio* appeared, in which he described the advantages of logarithmic calculation and attached the first tables.

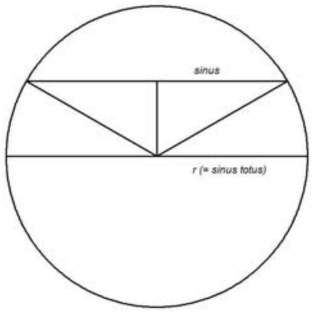
Napier initially used the term “artificial numbers”, then gave them the name *logarithm* in the sense of a ratio – formed from the Greek words *logos* (meaning: ratio) and *arithmos* (number); because the logarithms are defined by a numerical ratio:

$$a : b = c : d \quad \Leftrightarrow \quad \log(a) - \log(b) = \log(c) - \log(d)\,.$$

In order to be able to use the advantages of calculating with logarithms, the values of the underlying geometric number sequence should be as close as possible. In his *Arithmetica Integra* of 1544, Stifel considered the integer powers of 2; the distances between two consecutive numbers get bigger and bigger:

arithmetic sequence: $\log_2(x)$	-3	-2	-1	0	1	2	3	4	5	6
geometric sequence: x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64

Napier chose a geometric sequence with q = 0.9999999 and – in order to avoid decimal numbers – the circle radius $r = 10^7$ as the starting value for its logarithm table of sine values (at this time these were not aspect ratios in a right-angled triangle, but lengths of half circular chords, called *sinus totus*), i.e.



$g_n = 10^7 \cdot (1 - \frac{1}{10^7})^n$ and as an assigned arithmetic sequence

$a_n = n \cdot (1 + \frac{1}{2 \cdot 10^7})$ with $\log_{Nap}(10^7) = 0$.

The disadvantage is that for $x_1 < x_2$ the following applies: $\log_{Nap}(x_1) > \log_{Nap}(x_2)$ but above all:

If you want to perform a division $\frac{a}{b}$, it follows from $x = \frac{a}{b}$ and $\frac{a}{b} = \frac{x}{1}$ that:

$\log(x) - \log(1) = \log(a) - \log(b)$ so $\log(\frac{a}{b}) = \log(a) - \log(b) + \log(1)$.

Analogously it follows for the product: $y = a \cdot b$, that $\frac{y}{b} = \frac{a}{1}$ and the following applies:

$\log(y) - \log(b) = \log(a) - \log(1)$, i.e. $\log(a \cdot b) = \log(a) + \log(b) - \log(1)$.

The London mathematics professor Henry Briggs (1561 – 1630) recognized the arithmetic advantages that are possible through the use of "ratios" and took the arduous journey to Scotland to suggest to Napier that the number 10 should be used as the basis for the logarithms (which he had already considered in a previously written but only posthumously published work *Mirifici Logarithmorum Canonis Constructio*).

They also defined: log (1) = 0, which greatly simplified logarithmic calculations. During the following years, Briggs was busy calculating a table of 30,000 (out of a planned 100,000) logarithms with 14-digit accuracy. In his honour, decadal logarithms are still called *Briggs logarithms* today.

Briggs developed an ingenious method for this: He first (by hand) calculated the square root of 10, from this in turn extracted the square root, etc., over 50 times and determined each of the roots to 30 places.

Since every number x with $1 < x < 10$ can be represented as

$x = (10^{\frac{1}{2}})^{k_1} \cdot (10^{\frac{1}{4}})^{k_2} \cdot (10^{\frac{1}{8}})^{k_3} \cdot \ldots$ with $k_1, k_2, k_3, \ldots \in \{0,1\}$,

i.e. $\log_{10}(x) = k_1 \cdot 0.5 + k_2 \cdot 0.25 + k_3 \cdot 0.125 + \ldots$

the sequence of k_i indicates the sequence of digits of the logarithm of *x* in the binary system.

Example: For $x = 2$ one finds the following values for the k_i : 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, … , so

$\log_{10}(2) = \frac{1}{4} + \frac{1}{32} + \frac{1}{64} + \frac{1}{256} + \ldots \approx 0.30103$.

Shortly before his death, Napier completed his *Rabdologiae* (from the Greek ράβδος = sticks), in which he introduced special calculation rods (also known as Napier's bones, as they were often made of ivory).

These bars contain the multiples of the numbers from 0 to 9, with the "ones" digit at the bottom right and the "tens" digit at the top left.

	0	1	2	3	4	5	6	7	8	9
1	0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9
2	0/0	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8
3	0/0	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7
4	0/0	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6
5	0/0	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5
6	0/0	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4
7	0/0	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3
8	0/0	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2
9	0/0	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1

	7	3	0	3	4
1	0/7	0/3	0/0	0/3	0/4
2	1/4	0/6	0/0	0/6	0/8
3	2/1	0/9	0/0	0/9	1/2
4	2/8	1/2	0/0	1/2	1/6
5	3/5	1/5	0/0	1/5	2/0
6	4/2	1/8	0/0	1/8	2/4
7	4/9	2/1	0/0	2/1	2/8
8	5/6	2/4	0/0	2/4	3/2
9	6/3	2/7	0/0	2/7	3/6

For example, to multiply the numbers 73034 and 6, place the bars with multiples of 7, 3, 0, 3 and 4 next to each other; then you can read the result 438204 in the 6th line digit by digit from right to left:

4; 2 + 8 = 0 with carry 1; 1 + 1 + 0 = 2; 0 + 8 = 8; 1 + 2 = 3; 4.

For the multiplication of multi-digit natural numbers, the results of the multiplication of the individual digits of the second factor are noted, each offset by one place from one another.

For the product of decimal numbers, one must observe the rules of setting decimal points.

With the help of the NAPIER rods it is also possible to carry out division, in which the individual digits of the result are in principle guessed and then checked by multiplication. With a little practice, the rods can even be used for root extraction.

The method of multiplication with the help of the slide rule gave WILHELM SCHICKARD (1592 – 1635) suggestions for the development of a calculating machine, which he presented in 1623.

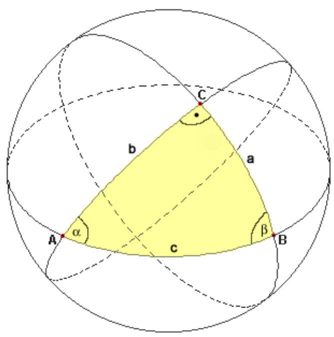
In 1608 SIMON STEVIN introduced a somewhat cumbersome notation for decimal numbers in Europe with his book *De Thiende*.

Example: 184⓪5①4②2③9④ means 184.5429.

JOHN NAPIER improved it in his last publications by using the decimal point to mark the boundary between the whole number and the fraction. This was then adopted in (almost) all countries.

Finally his merits in spherical trigonometry are to be mentioned. Even today, a set of formulas is referred to as NAPIER's rules (NAPIER was the first to recognise a common principle for the formation of the formulas):

In the right-angled spherical triangle, the cosine of each "part" is equal to the product of the cotangent of the adjacent parts and also the product of the sine of the non-adjacent parts (*where the "parts" are* $a, c, \beta, 90° - a, 90° - b$).



Example:

$\cos(c) = \sin(90° - a) \cdot \sin(90° - b) = \cos(a) \cdot \cos(b); \cos(c) = \cot(\alpha) \cdot \cot(\beta)$.

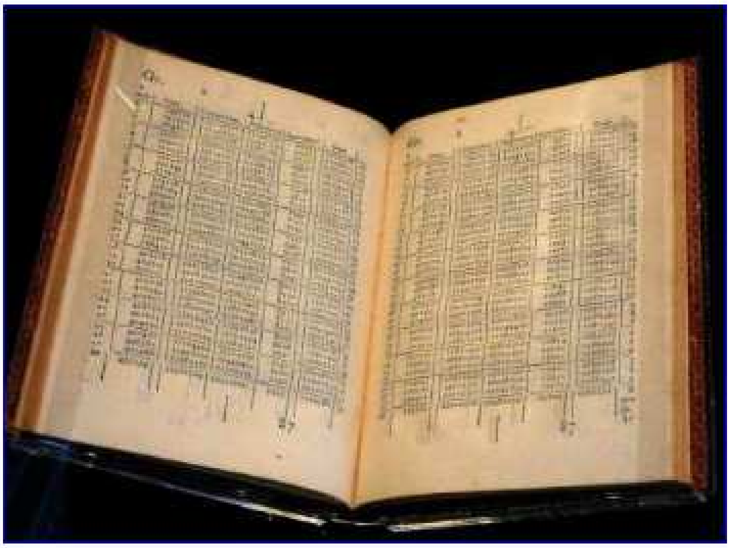
In addition, certain theorems about the sum or difference of sides and angles in the general spherical triangle are called *NAPIER's analogies*.

First published 2010 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/john-napier-1550-1617/1047087

Translated 2020 by John O'Connor, University of St Andrews

400 year celebration of Napier's logarithms



Merchiston castle

Some impressions from the Napier exhibition in Edinburgh in 2014 (© John O'Connor)

Here an important hint for philatelists who also like individual (not officially issued) stamps.
Enquiries at europablocks@web.de with the note: "Mathstamps".