Henri Poincaré (April 29, 1854 – July 17, 1912)

by HEINZ KLAUS STRICK, Germany

Already during his school years, the young JULES HENRI POINCARÉ, son of a professor of medicine at the University of Nancy (today called the *Université HENRI POINCARÉ* in his honour), attracted attention because of his remarkable mathematical talent, which he exhibited by several times winning the first prize in the national *Concours général*. During his university years at one of the elite universities of France, the *École Polytechnique* in Paris, one of his teachers was CHARLES HERMITE (who in 1873 achieved international recognition for his proof of the transcendence of the base of the natural logarithm *e*, that is, he proved that the number *e* is the root of no algebraic equation).

POINCARÉ completed his studies in two years, during which time he wrote his first scientific article. Then, out of interest in the practical applications of mathematics and the natural sciences, he

attended the country's most renowned engineering college, the *École des Mines*, where he completed a degree as an engineer and was then appointed to the post of mining engineer and inspector of mines. During this time, he also wrote his doctoral dissertation in mathematics on a particular class of differential equations. After a short period as lecturer in mathematics at the University of Caen (Normandy), he was appointed professor of mathematical physics and probability theory at the Sorbonne, in Paris.



(drawing: © Andreas Strick)

The constantly changing topics of his lectures demonstrate the broad spectrum of his areas of interest; they range over all areas of mathematics, astronomy, and mathematical physics. His lectures were frequently oversubscribed at the outset, but the audience soon diminished, since few students could keep up with the difficulty of his presentation.

By the time of his death at age 58 (from complications following surgery), he had published over five hundred scientific articles and books on a wide variety of topics, including some contributions to popular science, on account of which he was elected in 1909 to the *Académie Française*, thereby becoming one of the forty "immortals."

The amount of work that POINCARÉ took on was apparently limitless. While he did not become involved in politics (in contrast to his cousin RAYMOND, who assumed the office of prime minister five times and from 1913 to 1920 served as president of the republic), he became involved in the DREYFUS affair, serving on a commission that condemned the utter inadequacy of the evidence used against ALFRED DREYFUS, a Jewish officer in the French army, at his trial for treason.

POINCARÉ served on countless committees, often as the chairman, for example in the Académie des Sciences. He was the editor of scientific journals such as the Bulletin astronomique and the Journal des mathématiques pures.

As chairman of the *Bureau des Longitudes*, he worked on a standardization of the measurement of time (his suggestion was that non-metric units of time as well as the division of the circle into 360 degrees be replaced by decimal units, a proposal that was not accepted).

He took a personal interest in organizing expeditions to Peru to improve on the meridian-length data obtained in 1736 by LA CONDAMINE. Also, his work as a mine inspector continued for all of his life; indeed, he served as head of a commission that investigated a tragic mining accident.





POINCARE's workday proceeded according to a fixed plan. From 10 until 12 in the morning and from 5 until 7 in the afternoon, he devoted himself to research. In the evening, he studied the professional journals. When POINCARE began work on an article, he did not necessarily know whether and how he would end it. If the work began easily, then he would continue writing effortlessly and allow nothing to divert his attention.

But if the beginning of an article would not fall properly into line, he would stop work on it because he found the necessary "intuition" lacking. He was convinced that his subconscious mind would continue working on the problem until the time came that it would reappear – solved – in his conscious mind. Therefore, he did not work on his own problems during the evening, because he feared that his sleep might suffer if he did.

In 1906, he gave a lecture at a psychology convention in which he described how some of his inspired ideas had matured: conscious thinking, unconscious thought (incubation), inspiration, and validation.

POINCARÉ was convinced that (mathematical) logic had no part in the development of ideas, that it in fact limited the possibility of generating ideas. He opposed the attempts of contemporary mathematicians such as GIUSEPPE PEANO and DAVID HILBERT to describe all of mathematics through a system of axioms, suspecting the limitation of such an approach, a limitation that was in fact revealed in 1931 by the Austrian logician KURT GÖDEL.



Beginning with his doctoral dissertation, POINCARÉ worked on problems in complex analysis, investigating general properties of mappings as well as the structures that are preserved under certain mappings. In this connection, he introduced, for a particular class of mappings (in ignorance of prior work of FELIX KLEIN, at the time a professor of geometry at the University of Leipzig), the name FUCHSian functions (after the German mathematician LAZARUS FUCHS), even though the name KLEINian functions would have been more appropriate.

The exchange of letters on this topic between POINCARÉ and KLEIN that followed was not carried out in a cordial atmosphere. When POINCARÉ eventually discovered connections between these functions and non-EUCLIDian geometry, which was in fact *the* special area of knowledge to which FELIX KLEIN laid claim, this led to the latter's suffering a collapse (or perhaps the "victory" of a rival was just the trigger of a creative crisis for FELIX KLEIN during a phase of extreme overwork).



Both KLEIN and POINCARÉ independently developed models of so-called hyperbolic geometry, in which – in contrast to EUCLIDian geometry – for a given line and point external to the line, there are at least two lines that pass through the point and are parallel to the line. In one of the models developed by POINCARÉ, the "lines" are modelled by circular arcs and diameters that run perpendicular to the boundary (a circle); in the other, it is semicircles and half-lines that are perpendicular to the boundary line.

In 1885, King OSCAR II OF SWEDEN underwrote a prize competition. The commission, which comprised the Swedish mathematician MAGNUS GUSTAF (GÖSTA) MITTAG-LEFFLER, the German KARL WEIERSTRASS, and CHARLES HERMITE, quickly concluded that the 160-page contribution from POINCARÉ on the three-body problem should be awarded the prize, even though he had not completely solved the problem that had been posed. (In the three-body problem, one considers the orbits of celestial bodies whose gravitational forces mutually interact and attempts to describe them mathematically. The integrals that arise cannot be solved using elementary functions. Special cases had been considered by JOHANNES KEPLER and JOSEPH-LOUIS LAGRANGE.)



Part of the prize was publication of the winning essay. In this regard, POINCARÉ was asked to clarify some points. He did so at nine places, but in the course of reviewing his work – by this time, the paper had been printed and distribution had begun – he noted that at an important place, there was an error that could not be easily corrected. The upshot of all this is that our solar system is unstable, in the sense that small changes in the initial conditions can have a massive effect on the outcome. This discovery marks the beginning of what today is known as chaos theory.

POINCARÉ had to assume the costs of reprinting the article, which in the meantime had grown to 270 pages, costs that ultimately exceeded the amount of the prize money, while MITTAG-LEFFLER saw to it that the copies that had already been distributed were collected and destroyed.

POINCARÉ was constantly being criticized that he did not work with sufficient attention to detail, that his argumentation was erratic and contained gaps. He replied with the assertion that his conclusions were *évident* and that he had no time to work out all the details, simply because he had too many ideas floating about in his head.

POINCARÉ is considered the founder of *algebraic topology*. Using algebraic methods, one investigates properties of geometric objects – such as stretching, compressing, and twisting – that remain unchanged under certain types of mappings, called *homeomorphisms*.

While his predecessors concentrated their investigations on two-dimensional manifolds (surfaces) such as the sphere and the torus, POINCARÉ studied three-dimensional manifolds. His conjecture that every *simply connected compact three-dimensional manifold is homeomorphic to the surface of the four-dimensional sphere*, that is, the 3-sphere, became one of the most famous unsolved problems in the history of mathematics.

Finally, in 2002, GRIGORI PERELMAN came up with a proof. He was awarded a million-dollar prize for having solved one of the so-called *millennium problems*, but he refused the award, as he did likewise the award of the FIELDS Medal (known popularly as the "NOBEL Prize of mathematics").



POINCARÉ received many academic distinctions from foreign universities and was named a member of many learned societies including the Royal Society.

He was also repeatedly put forward for the Nobel Prize in physics, though he did not receive that award, in contrast to HENDRIK ANTOON LORENTZ and ALBERT EINSTEIN.

At the end of the nineteenth century, many scientists were concerned with the question how a worldwide network of clocks could be synchronized using electromagnetic signals. To solve the problems of motion of electromagnetic waves in the hypothetical ether surrounding the Earth (whose existence apparently derived from *MAXWELL's equations*), LORENTZ developed a mathematical model that POINCARÉ called *LORENTZ transformations*.

In 1905, POINCARÉ presented a paper to the *Académie des Sciences* that contained the mathematical foundations for the special theory of relativity; this was several weeks before publication of ALBERT EINSTEIN's famous paper on the subject. In contrast to EINSTEIN, POINCARÉ did not attempt (for pragmatic reasons, as he put it) to replace classical mechanics with a new theory, but instead to generalize the MAXWELL-LORENTZ electrodynamics taking the ether into account.

EINSTEIN, in contrast, began with the principle that the speed of light in vacuum is constant, and he went on to develop a theory that is valid without assuming the existence of the ether. EINSTEIN and POINCARÉ met each other only once, during a conference in 1911.

By the time of his death the following year, POINCARÉ had received not a word of recognition for his work leading to EINSTEIN's theory. It was not until the 1950s that EINSTEIN himself had something to say about Poincaré's contribution to the special theory of relativity.



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