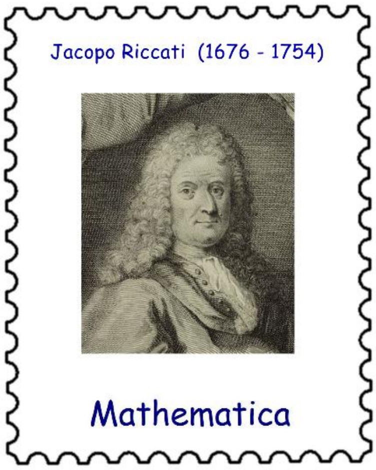
Jacopo Riccati (March 28, 1676 – April 15, 1754)

by HEINZ KLAUS STRICK, Germany



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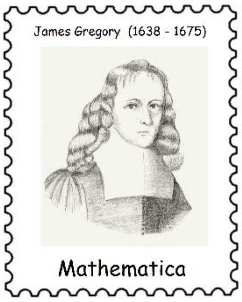
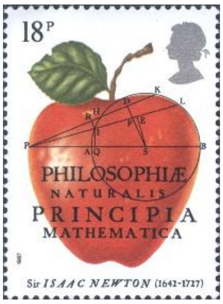
JACOPO FRANCESCO RICCATI was born as the son of Count MONTINO RICCATI who was born in the Republic of Venice; his mother GIUSTINA came from the old Roman noble family of the COLONNA. When JACOPO was 10 years old, his father died.

One of his father's brothers then made sure that the obviously gifted boy received a suitable education and for the next six years, JACOPO – like many other children from noble families – attended the Jesuit college in Brescia, about 180 km from Venice. RICCATI seems to have had fond memories of the college, as later on he also sent his own sons there.

In 1693, JACOPO RICCATI began studying law at the University of Padua, in keeping with family tradition, but he was also interested in other subjects. He was particularly fascinated by the advances in astronomy, so he attended the lectures of STEPHANO DI ANGELI. He was one of the most committed defenders of the *method of indivisibles* of his teacher BONAVENTURA CAVALIERI. In the 1660s, JAMES GREGORY was also one of DI ANGELI's students.





When in 1687 ISAAC When NEWTON's *Philosophiae Naturalis Principia Mathematica* was published, DI ANGELI became aware that a new era in mathematics had begun with the development of infinitesimal calculus. He gave his copy of the *Principia* to the young RICCATI, with whom he had become friends.

In June 1696, JACOPO RICCATI completed his law studies and a few months later he married the noblewoman ELISABETTA DI CONTI D'ONIGO. The happy marriage produced 18 children – only half of whom reached adulthood. Two of his sons had careers as scientists: VINCENCO and GIORDANO (see below).

JACOPO RICCATI lived with his family on his inherited lands in Castelfranco Veneto (about 35 km north of Padua). At times he took over the office of mayor of the municipality and he was repeatedly asked by the Senate of the Republic of Venice for advice on the construction of canals and dykes.

JACOPO RICCATI devoted his free time to studying current publications in various sciences; he regularly read the *Commentari dell'Accademia delle Scienze di Bologna,* the journal *Acta Eruditorum* published in Leipzig, the publications of the *Russian Academy of Sciences* in Saint Petersburg and the *Giornale de' Letterati d'Italia*.

In 1712, the *Giornale*, which also published some of RICCATI's contributions on philosophical and literary topics, published a mathematical problem as a challenge to readers:

- *What we are looking for is the equation of a curve whose radius of curvature depends only on the coordinates of the individual points.*

RICCATI was able to show that the question leads to a special second-order differential equation, for which he gave a solution procedure.

Now RICCATI was a man in demand: the Russian Tsar PETER THE GREAT offered him the position of President of the *Academy of Sciences* in Saint Petersburg, the University of Padua offered him a chair in mathematics; the Habsburg court also tried to lure him to Vienna as an imperial advisor.

RICCATI rejected all of these offers – he did not want any changes in his previous quiet life with his large family. He could not be tempted with money either – his wealth was enough for his relatively modest needs. He travelled little, only once going to Val di Sole for a health cure, where he met NICOLAUS (II) BERNOULLI, with whom he exchanged intensive ideas about solving differential equations.

RICCATI had been working on the subject of *differential equations* since 1707, when the mathematician GABRIELE MANFREDI, who taught in Bologna, published the first book on this new branch of mathematics. (*De constructione aequationum differentialium primi gradus*).

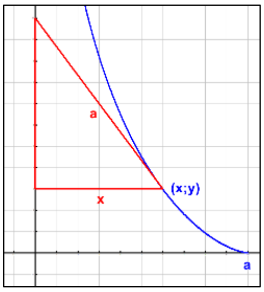
An equation for a function with one variable, in which derivatives of this function also occur, is called an *ordinary differential equation* (ODE) – in contrast to a *partial* differential equation, in which several function variables and *partial* derivatives can occur.

The *order* of a differential equation is determined by the order of the highest occurring derivative. Solving a differential equation means determining a suitable function that satisfies this equation. In general, the solution is a *set* of functions and by specifying a so-called *initial condition,* the number of solutions can be reduced.

ISAAC NEWTON formulated the first differential equation in 1671:

$y' = 1 - 3x + y + x^2 + xy$ (in today's notation) with the initial condition $y(0) = 0$. He solved it using the approach of a series expansion $y(x) = a_0 + a_1 x + a_2 x^2 + \ldots$ and by comparing coefficients.

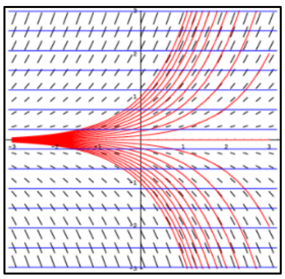
GOTTFRIED WILHELM LEIBNIZ described the process of pulling a pocket watch on a chain across a tabletop (the so-called *drag curve,* see the figure on the right) using the ODE $y' = \frac{dy}{dx} = -\frac{\sqrt{a^2-x^2}}{x}$. This ODE is of the simple type $y' = f(x)$; it has the solution $y = a \cdot \ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right) - \sqrt{a^2 - x^2} + C$ where $y(a) = C = 0$.



LEIBNIZ considered the integrals of the two sides of the equation $dy = -\frac{\sqrt{a^2-x^2}}{x} dx$, i.e. $\int dy$ and $-\int \frac{\sqrt{a^2-x^2}}{x} dx$.

Separate integration is possible here because the variables are on different sides of the equals sign; otherwise one must try to achieve a *separation of the variables* (*separatio inderminatarum*, a term coined by JACOB BERNOULLI in 1694).

In the 1690s it was mainly the brothers JACOB and JOHANN BERNOULLI, who developed various methods for solving ODEs, including the visualisation by so-called *direction fields*.

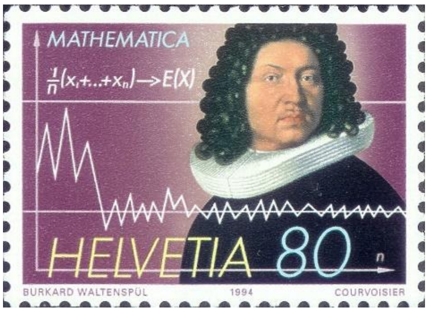


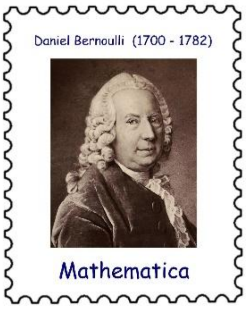
The Wikipedia figure on the right (by *jjbeard*) shows the direction field for the ODE $y' = y$, i.e. $\frac{dy}{y} = dx$, which leads to a family of exponential functions $\ln(y) = x + C$.

BERNOULLI equations, named after the two brothers, generally have the form
$y' + P(x) \cdot y = Q(x) \cdot y^n$.

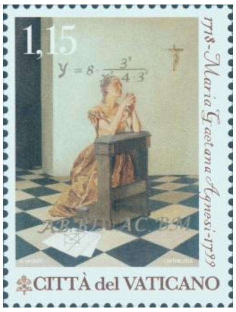
To solve such a differential equation, if $n = 0$, one can use the trick of substituting $y' = y$, i.e. $y' = u' \cdot v + u \cdot v'$. In the case $n = 1$ the separation of the variables is achieved immediately, in case $n > 1$ one can proceed with substitution $u = y^{1-n}$.

ODEs of the form $y' = P_0(x) + P_1(x) \cdot y + P_2(x) \cdot y^2$ are today called RICCATI ODEs. The name comes from DANIEL BERNOULLI, the son of JOHANN BERNOULLI, who in 1724 (when he was only 24 years old) was able to give a general solution procedure for ODEs of this type based on the paper by RICCATI mentioned below.





RICCATI, who never aspired to a professorship, wrote a 154-page paper *Delia separazione delle indeterminate nelle equazioni differenziali di prima e di secondo grado, e della riduzione delle equazioni differenziali del secondo grado e d'altri gradi ulteriori* (On the method of separating variables in first- and second-order differential equations and the reduction of higher-order differential equations) in the early 1720s which was published in stages. He used it to promote gifted students, including RAMIRO RAMPINELLI (1697-1759, later professor of mathematics in Rome and Bologna), to whom he gave private lessons, and MARIA GAËTANA AGNESI (1718-1799), who expressed her heartfelt thanks for RICCATI's generous support in the prologue to her book *Instituzioni analitiche ad Uso della Gioventù Italiana* (Textbook of Analysis for Italian Youth) .



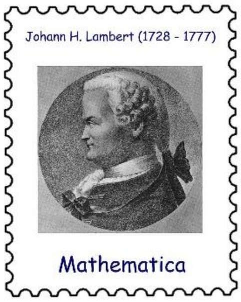
After the death of his wife in 1749, RICCATI moved to a house in Treviso, 25 km away, which was also owned by the family.

There he died at the age of almost 78, after – it is said – a feverish illness.

He was buried in a side chapel of the cathedral in Treviso that belonged to the family. A few years after his death, his sons VINCENZO and GIORDANO published his works in four volumes.

VINCENZO, the second son of JACAPO RICCATI, joined the Jesuit order at the age of 19. After completing his studies, he taught mathematics and physics at the Jesuit college in Bologna for over 30 years. He continued his father's work on the subject of differential equations; in addition, he dealt with physical questions related to force parallelograms and the principle of energy conservation.

Independently of JOHANN HEINRICH LAMBERT, he introduced the hyperbolic functions *sinh*, *cosh* and *tanh*.



VINCENZO RICCATI was one of the first members of the *Accademia Nazionale delle Scienze*, founded in 1782 and he was also an honorary member of the *Russian Academy of Sciences* .

The research focus of GIORDANO, the fifth son of JACAPO RICCATI, was more in the field of music and technology. Among other things, he worked on the elasticity of metals – 25 years before the physicist THOMAS YOUNG.

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https://www.spektrum.de/wissen/jacopo-riccati-differenzialgleichungen-und-ein-ruhiges-leben/2246353

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