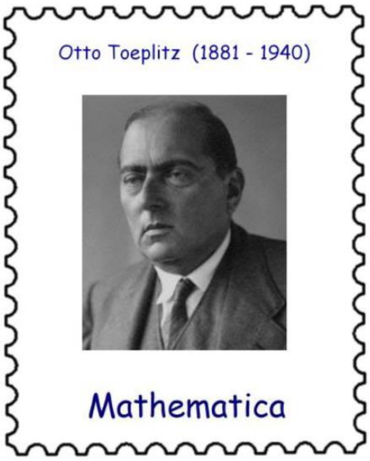
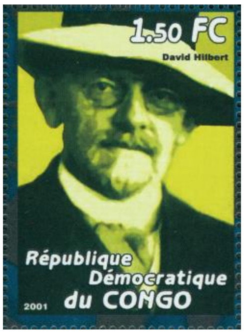
OTTO TOEPLITZ (August 1, 1881 – February 15, 1940)

by HEINZ KLAUS STRICK, Germany



When OTTO TOEPLITZ was born in Breslau (now Wrocław in Poland) in 1881, his future career was almost predictable since both his grandfather JULIUS and his father EMIL had worked as mathematics teachers at grammar schools in Breslau and Lissa (Posen district) respectively and both had also published articles on mathematics teaching. EMIL TOEPLITZ was also known throughout the German Reich as the editor of the annual *Philologenjahrbuch* (KUNZE's Calendar), a directory of all teachers working at grammar schools and similar institutions (which still exists today).



After passing his *Abitur* examinations, OTTO TOEPLITZ began studying mathematics at the University of Breslau.

In 1905 he completed his doctorate on a topic from algebraic geometry (*On the transformation of sets of bilinear forms of infinitely many variables*). In 1907 he *habilitated* and became a private lecturer.

Inspired by DAVID HILBERT, he worked intensively on the theory of integral equations, on which he wrote several papers and later also an encyclopaedia contribution for the *Encyclopaedia of the Mathematical Sciences*.

e	*f*	*g*	*h*	*i*
d	*e*	*f*	*g*	*h*
c	*d*	*e*	*f*	*g*
b	*c*	*d*	*e*	*f*
a	*b*	*c*	*d*	*e*

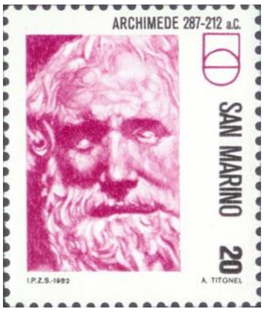
In 1911 TOEPLITZ published a paper on systems of equations whose coefficient matrix is symmetrical with constant elements in each descending diagonal. In the case of finite systems, therefore, only at most $2n-1$ instead of n^2 different coefficients occur and the solution procedures certainly simplified considerably. Matrices of this type are called TOEPLITZ matrices.

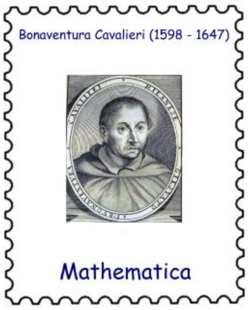
In 1913 TOEPLITZ accepted a position as associate professor at the *Christian Albrechts University* in Kiel and in 1920 the position was converted into a full professorship. He was passionate about his teaching and set up an educational colloquium for his student teachers, which dealt in particular with topics from the history of mathematics.

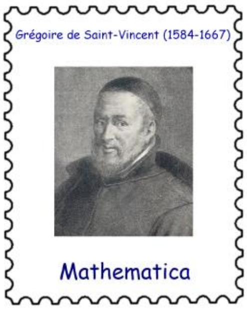
In 1926, at the annual conference of the *Society of German Natural Scientists and Physicians* in Düsseldorf, TOEPLITZ gave a much-acclaimed lecture on the teaching of calculus, in which he advocated having the students trace the historical development of calculus (the so-called "genetic method"):

> *...Mathematics and mathematical thinking are not only part of a special science, but are also closely connected with our general culture and its historic development of mathematical thinking, a bridge to the so called Arts and Sciences and the seemingly so non-historic exact sciences can be found ... Our main purpose is to help build such a bridge. Not for the sake of history but for the genesis of problems, facts and proofs, for the sake of the decisive turning points of that genesis...*

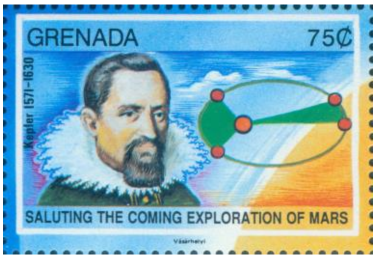
In order to realise this project, TOEPLITZ planned a two-volume work, but was no longer able to put it into practice. In 1949, his materials were compiled by his former assistant GOTTFRIED KOETHE and published posthumously as a book entitled *Die Entwicklung der Infinitesimalrechnung – eine genetische Annäherung* (The Development of the Infinitesimal Calculus – a Genetic Approach), consisting of the three chapters: The nature of the infinite process; The definite integral; Differential and integral calculus.







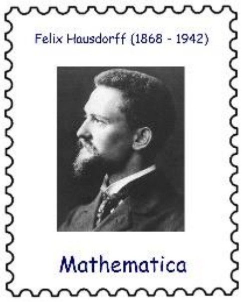
Because of the lack of previous knowledge of the first-year students, TOEPLITZ recommended that the concept of limit value should only be tackled exactly at a later point in time, and that the integral calculus should also be treated before the differential calculus – in accordance with the historical development (ARCHIMEDES, CAVALIERI, FERMAT, SAINT VINCENT). The book ended with explanations of KEPLER's laws.



His great interest in historical context led to the founding of the journal *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* (Sources and Studies on the History of Mathematics, Astronomy and Physics) in 1926 (together with OTTO NEUGEBAUER and JULIUS STENZEL).

In 1928 TOEPLITZ accepted an appointment to Bonn, where he had better working opportunities than in Kiel and where a larger number of students were also enrolled.

At Bonn University he made friends with FELIX HAUSDORFF.



Together with his assistant GOTTFRIED KÖTHE he developed his own theory of infinite-dimensional spaces, as the theory of the Polish mathematician STEFAN BANACH seemed too abstract to him.

TOEPLITZ also had a lively exchange with the Münster professor HEINRICH BEHNKE and from 1932 onwards the *Mathematical-Physical Semester Reports* were published, which were aimed in particular at mathematics teachers. They are still published today.

After the *Law for the Restoration of the Professional Civil Service* of 1933 came into force, professors of Jewish origin were initially able to continue their teaching activities, as an exception was made for those who had already worked as university teachers before 1914.

This regulation was abolished in 1935 by the Nuremberg Laws, and TOEPLITZ was forced to retire against his will. TOEPLITZ, who until then had hardly observed Jewish laws, took over the office of head of the Jewish community in Bonn. He founded a school for Jewish children, where he also took on teaching himself.

As head of the university department in the *Reichsvertretung der Juden in Deutschland* (Reich Representation of Jews in Germany), he arranged scholarships for particularly gifted Jewish students and organised their departure for the USA.

When the number of suicides in his environment increased dramatically and he himself no longer felt able to cope with the pressure from the National Socialists, he emigrated to Palestine (then under British Mandate) in February 1939.

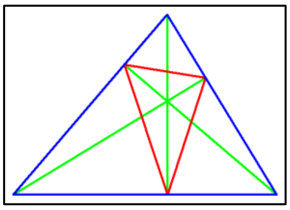
There he immediately became involved in building up the Jewish university on Mount Scopus in Jerusalem; but a year after his arrival he fell seriously ill and died.

TOEPLITZ was described by his students – as well as by his colleagues – as a friendly and helpful person who always took time for others.

In 1930, a collection of popular topics from his lectures was published: the book *Von Zahlen und Figuren – Proben mathematischen Denkens für Liebhaber der Mathematik* (Of Numbers and Figures – Samples of Mathematical Thinking for Mathematics Lovers), which is still worth reading today. This was produced with the collaboration of HANS RADEMACHER, Chair of Mathematics in Breslau. The pacifist RADEMACHER had to emigrate in 1934.

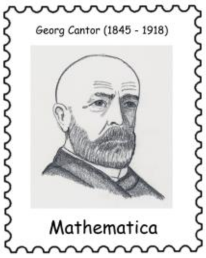
In 22 sections, the two authors attempted to break through the wall that separates non-mathematicians from the world of mathematicians, beginning with the ingenious idea of EUCLID's proof of why there are infinitely many prime numbers.

Next, aspects of how to develop an optimal routing in a tram-rail network were explained.



In the third section, it is proved that among all the *n*-polygons inscribed in a circle, the regular one has the largest area.

The proof of the irrationality of 2 is followed by two illustrative proofs that among all triangles that can be inscribed in a triangle, the triangle whose vertices are the bases of the altitudes has the smallest circumference.

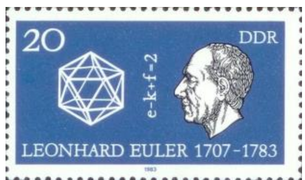


In chapter 8, TOEPLITZ goes into CANTOR's considerations on the power of sets and addresses the continuum problem.

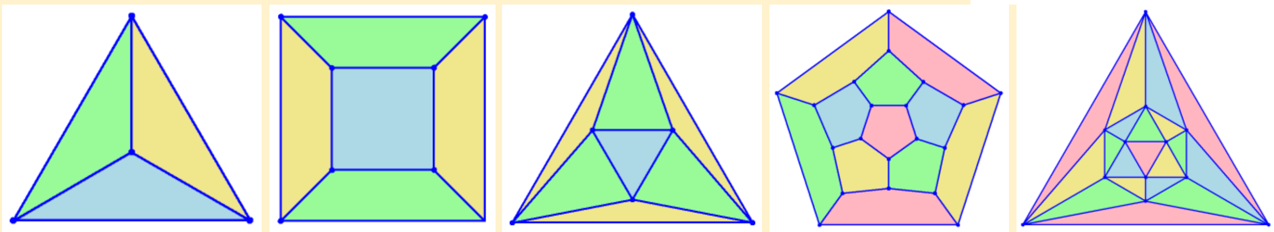
The following chapter deals with intersections on the straight circular cone, followed by a section on WARING's problem for n = 2, 3, 4: *Every natural number can be represented as a sum of at most $g(n)$ powers with exponent n, where* $g(2) = 4$; $g(3) = 9$; $g(4) = 19$.

In section 10 TOEPLITZ deals with double points of closed, self-interpenetrating curves.

In chapter 11 it is shown that the decomposition of natural numbers into prime factors is unique (with a digression into sets of numbers whose elements can be represented in the form $a+b\cdot\sqrt{6}$ or $a+b\cdot\sqrt{-6}$).



Chapter 12 introduces the four-colour problem (proven in 1976) and EULER's polyhedron theorem and explains connections between them.

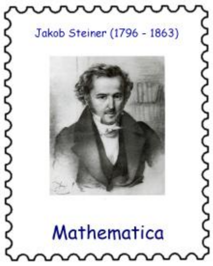


Chapter 13 is devoted to the statement of FERMAT's conjecture (still unproven in 1930). First it was explained how to find all *Pythagorean number triples* in the case *n* = 2, then why the equation $x^4 + y^4 = z^4$ has no solutions in $\mathbb{N}$.

The next section deals with the question of the maximum radius of a circle in which all points of a given collection of points lie.

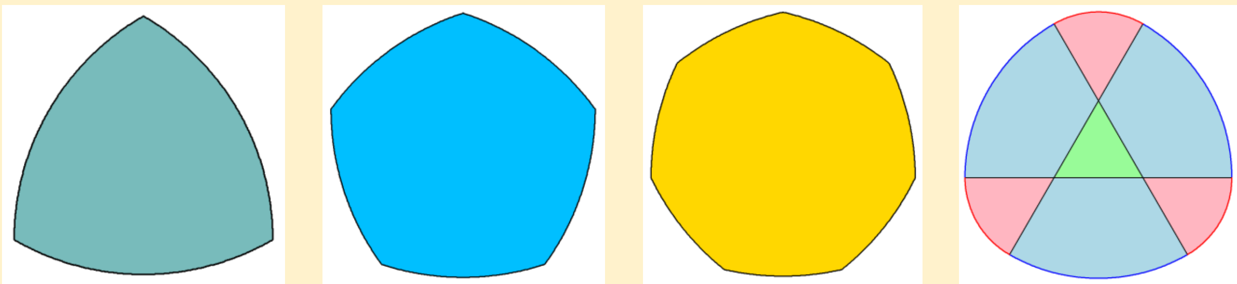
The 15th section deals with the approximation of irrational numbers by rational numbers; among other things, it is shown that $0 < \sqrt{2} - \frac{7}{5} < \frac{1}{5^2}; 0 < \frac{17}{12} - \sqrt{2} < \frac{1}{12^2}; 0 < \sqrt{2} - \frac{41}{29} < \frac{1}{29^2}; \ldots$

In chapter 16, straight line guidance by joint mechanisms is examined, and in the next section it is explained what EUCLID and EULER have found out about the perfect numbers.



Then it is described why, for a given circumference, the circle is the figure with the largest area (proof idea according to JACOB STEINER).

Chapter 19 deals with periodic decimal fractions, Chapter 20 with curves of constant width.



The penultimate chapter is devoted to the question of constructability with compass and ruler, and for which constructions the compass or the ruler could be dispensed with.

Finally, TOEPLITZ deals once again with prime numbers and their growth.

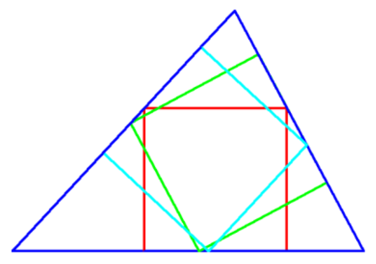
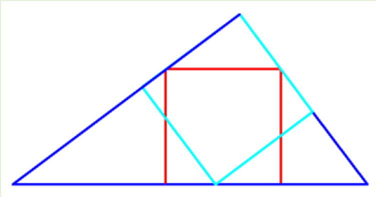
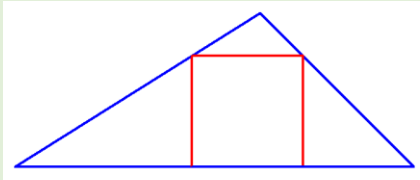
It is shown that the number 30 is the largest number for which it is true that all numbers below it that are not divisors of it are prime numbers.
The following applies to the sequence of prime numbers:
$2^2 > 1; 3^2 > 2; 5^2 > 2 \cdot 3 = 6; 7^2 > 2 \cdot 3 \cdot 5 = 30$,
but then the direction of the inequality sign changes:
$11^2 < 2 \cdot 3 \cdot 5 \cdot 7 = 210; 13^2 < 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2310; \ldots$

OTTO TOEPLITZ made an assumption in 1911, which has been proven for many types of curves, but not yet in general:

- *A square can be inscribed in every closed JORDAN curve C* (i.e. a continuous, non-intersecting plane curve).

Examples:

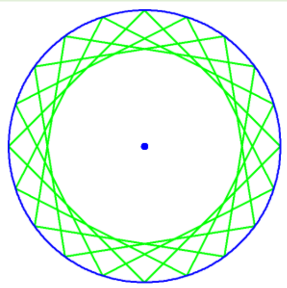
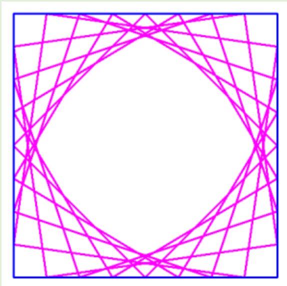
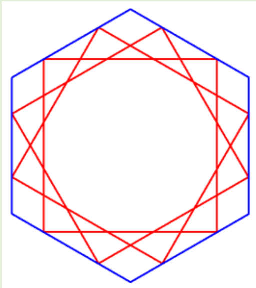
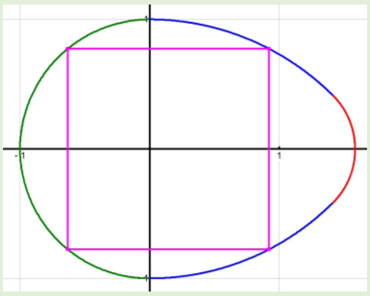
If *C* is a triangle, then – depending on whether it is an obtuse-angled, a right-angled or an acute-angled triangle – one or two or three squares can be drawn whose vertices lie on the circumference line.

If *C* is a circle or a square, then an infinite number of squares can be drawn,

if *C* is a regular hexagon, then three squares,

if *C* is an oval, then one square.

First published 2022 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/otto-toeplitz-der-engagierte-lehrer/1959523

Translated 2022 by John O’Connor, University of St Andrews
