Jurij Vega (March 23, 1754 – September 26, 1802)

by Heinz Klaus Strick, Germany

That their son would one day bear the name Georg Freiherr von Vega (Latinised: Georgius Bartholomaei Vecha) was something the parents – simple farmers in the Krainer Land – could certainly never have imagined.

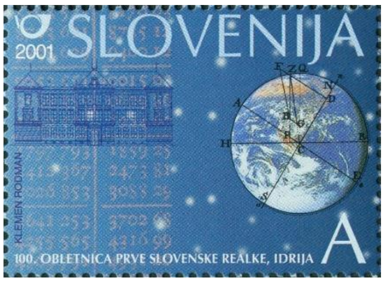
The father died when Jurij was six years old, and it was uncertain whether the child would one day be able to attend the Jesuit Latin School in nearby Ljubljana (then: Province of *Inner Austria* of the Habsburg Empire, today: Slovenia).

However, this school attendance was very successful. Jurij Vega finished school at the top of his class, not least thanks to the support (also financial) of his mathematics teacher, who recognised the boy's special mathematical talent. Later Jurij Vega remembered his patron and, out of gratitude, dedicated the second edition of his successful *Tafelwerk* [Handbook] to him.

At the age of 21 he found a job as a navigation engineer and worked on the regulation of the Sava, Slovenia's largest river. In 1780 he joined the military service and quickly made a career there. After attending the officers' school in Vienna, he soon received an appointment as *Teacher of mathematics at schools of the Austrian artillery corps*.

And since he was not satisfied with the existing teaching materials, he himself wrote the instructional work *Rechenkunst und Algebra* [Arithmetic and algebra] the first volume of the *Lectures on mathematics*, which appeared in 1782 in a comparatively high initial print run of 1500 copies. The book was so successful, among other things, because it was written in comprehensible language and thus contributed to the fact that even the poorly educated gunners and non-commissioned officers could receive a basic mathematical education.

In 1784, 1788 and 1800, further volumes followed with topics from higher mathematics: planimetry, stereometry, plane and spherical trigonometry as well as the beginnings of differential and integral calculus (the "beginnings" comprised considerably more than is taught in schools today), mechanics of solid bodies and hydrodynamics. By 1850, the 4-volume work had been published seven times.



In 1783, Jurij (Georg) Vega published tables with 7-digit base-10 logarithms, which surpassed all other tables available until then in terms of correctness. Due to the great sales success, the *Logarithmic-trigonometric Handbook* followed in 1793 – with the subtitle: *Instead of the small Vlackian, Wolfian and other similar, mostly very erroneous, logarithmic-trigonometric tables, designed for the mathematically inclined*.

On almost 400 pages, the work covered the logarithms of the natural numbers from 1 to 101000 as well as the logarithms of the trigonometric functions for angles between 0° and 45° with a step size of 10 arc seconds.

The 40-page preface, written in Latin and German, explained in detail how calculations could be carried out with the help of logarithms and how missing values could be found by interpolation.

The appendix contained an overview of all possible problems for plane and spherical triangles with the formulas needed to solve them. In later editions there were also lists of prime numbers and much more.

The subtitle referred to the logarithm tables of Adriaen Vlack (Vlacq), which had sold successfully until then. Together with Ezechiel de Decker, he was the first to publish a table with 10-digit base-10 logarithms of the natural numbers from 1 to 100000 in 1628, thus achieving the goal originally set by Henry Briggs.

Vega took over the logarithm values of his predecessors, but checked them conscientiously. In doing so, he also made use of the soldiers under his command by promising them a reward of one gold ducat for each error discovered. It is said that he only had to spend two ducats by the time of publication.

The sales success was again enormous; even many years after his death the work was reprinted and edited (in 1856 the 75th, in 1970 the 100th edition), also in English, French, Italian, Russian and Dutch.

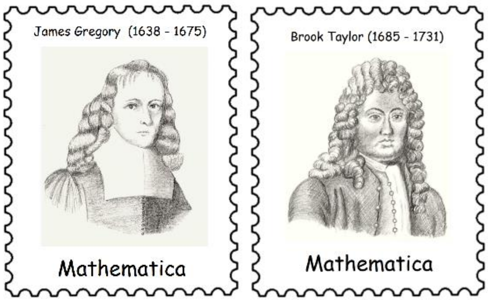
In 1794 Jurij Vega published a third set of logarithmic tables, the *Thesaurus logarithmorum completus*. The 10-digit logarithms it contained were intended especially for calculations in astronomy.

It is said that Carl Friedrich Gauss criticised the incorrectness of several values in the 10th decimal place.

In fact, errors in the last digits were discovered again and again later and corrected in subsequent editions. However, the tremendous progress in engineering and astronomy would have been inconceivable without Vega's tables.

Since Briggs's ingenious approaches, the method of calculating logarithms had changed completely due to the rapid development of differential calculus:

James Gregory (1638-1675) and Brook Taylor (1685-1731) had discovered the possibility of representing functions with the help of their derivatives (Taylor's theorem).



(drawings © Andreas Strick)

For natural logarithms (i.e. to base *e*), the following series expansion applies:

$$\ln(1+x)=\frac{x^1}{1}-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+-\ldots$$

From the formula $\ln(1-x)=-\frac{x^1}{1}-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\ldots$ we get a rapidly converging series:

$$\ln\left(\frac{1+x}{1-x}\right)=2\cdot\left(\frac{x^1}{1}+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\ldots\right)$$

If one wants to calculate, for example, $\ln(2)$ one first determines the corresponding value of *x*:

$\frac{1+x}{1-x}=2 \Leftrightarrow x=\frac{1}{3}$ and then calculates: $\ln(2)=2\cdot\left(\frac{1}{1\cdot 3^1}+\frac{1}{3\cdot 3^3}+\frac{1}{5\cdot 3^5}+\frac{1}{7\cdot 3^7}+...\right)$

With the first ten summands, the value is obtained to ten digits.

The following relationship now applies between the base 10 and the natural logarithms
$\lg(x)=\log_{10}(x)=\frac{\log_e(x)}{\log_e(10)}=\frac{\ln(x)}{\ln(10)}$.

To calculate the base 10 logarithm of a number *x*, the natural logarithm of *x* must therefore first be determined and this must then be multiplied by the reciprocal of the natural logarithm of 10:

$\frac{1}{\ln(10)}=\frac{1}{\ln(2)+\ln(5)}=0.4342944819...$

VEGA performed this calculation for all prime numbers up to 100,000 and deduced the logarithms of the others from $\log(a\cdot b)=\log(a)+\log(b)$.

In 1793, VEGA attracted the attention of the mathematical world with another brilliant calculation: he determined the number π to 136 decimal places. This record was not broken until 30 years later.

Here, too, he used the series expansion of a function: $\arctan(x)=\frac{x}{1}-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+-...$,

and also the addition theorem for the tangent: $\tan(a+b)=\frac{\tan(a)+\tan(b)}{1-\tan(a)\cdot\tan(b)}$.

From this he derived the following relationship: $\pi=20\cdot\arctan(\frac{1}{7})-8\cdot\arctan(\frac{3}{79})$,

which converges quickly because of the small arguments. He checked his calculations with
$\pi=8\cdot\arctan(\frac{1}{3})+4\cdot\arctan(\frac{1}{7})$.

As an officer, VEGA took part in several wars. After writing *Praktische Anweisung zum Bombenwerfen* (Practical Instructions on Bomb Throwing) in 1787, he enlisted the following year to take part in the siege of the city of Belgrade, which at that time still belonged to the Ottoman Empire.

Thanks to improved aiming accuracy, the attackers quickly succeeded in capturing the city.

During the wars against the French revolutionary armies, he improved the range of the cannons, which he had aligned according to mathematical-physical considerations (elevation angle 15°-16° as opposed to the previously usual 50°-75°).

In 1796, he received the *Order of MARIA THERESA*, the Habsburg monarchy's highest award for bravery, for his military service, and in 1800 he was raised to the hereditary peerage.

Because of his scientific achievements, VEGA was appointed a member of several institutions, including the *Prussian Academy of Sciences* in Berlin.

In September 1802, VEGA was reported missing and eventually his body was found in the Danube. At first it was assumed that an accident caused his death, but nine years later it turned out that VEGA had been the victim of a robbery and murder.

On the Slovenian stamps of 1994 and 2005, in addition to VEGA himself, the lunar crater VEGA is also depicted, including the graph of the natural logarithm function with $\ln(2) \approx 0.6931472$.

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